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Thermal Loss due to Non-Ideal Gas Behavior of Helium in VM Heat Pump

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Key Words : Helium, Real gas, VM heat pump, Heat pump

Abstract

A cycle analysis is performed to investigate how the non-ideal gas behavior of helium reduces the heating capacity of VM heat pumps. Since the operating pressures of VM heat pumps are as high as 1 to 20 MPa, the compressibility factor of helium becomes clearly greater than 1 and the non-ideal gas behavior always represents a thermal loss in heating. To calcutlate the amount of the losses, an adiabatic cycle analysis is performed with the real properties of helium and the net enthalpy flows through the two regerators are numerically obtained. It is shown that the non-ideal gas losses could be as much as 8% in the heating capacity when the operating pressures are greater than 10MPa. The effects of the operating temperatures and the dead volumes on the loss are presented.

Nomenclature

A_D : Mathematical derivative of mass in dead space with respect to pressure, defined by Eq.(6)
C_p : Specific heat at constant pressure
h : Specific enthalpy
h* : Specific enthalpy at inlet or exit of working volume
$\langle H \rangle$: Net enthalpy flow per cycle
m : Mass
$\dot{m}$: Mass flow rate
P : Pressure
Q : Heat per cycle
t : Time
T : Temperature
v : Specific volume
V : Volume
W : Work per cylce
Z : Compressibility factor

Greek symbols

α : Volume expansion coefficient
B : Isothermal compressibility

Subscripts

A : Warm working volume

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D : Dead volume(heat exchanger and regenerator)
H : Hot working volume
HX : Heat exchanger
L : Cold working volume
REG : Regenerator
TOT : Total

1. Introduction

Vuilleumier (VM) heat pump is a useful energy utility to perform both the cooling and the heating in residential or office buildings. Since the VM heat pump is driven by heat instead of the electrical work, it has a significant advantage of the peak saving of the electricity, especially as a cooler in the summer. At the same time, the typical working fluid of the VM heat pump is helium, which is a non-CFC refrigerant and is considered to be environmentally safe. On the other hand, the high operating pressure and the relatively low COP can be pointed as the disadvantages of the heat pump. To data, there have been active research works to examine the feasibility for the practical application of the VM heat pump in heating or cooling[(1)~(4)]

As shwon in Fig.1, the VM heat pumps is composed of three working spaces(hot : H, warm : A and cold : L), three heat exchanges (hot : HHX, warm : AHX and clold : LHX) and two thermal regenerators. The heat input from the hot temperature source causes the reciprocating motion of the displacers and the periodical oscillation of pressure, which results in the heat absorption from the cold outdoor and the heat rejection to the warm room. From the viewpoint of the energy flow, the VM heat pump can be compared with the normal heater as shown in Fig.2. The energy balance for the heat pump can be expressed such that the heat rejected to the room is the sum of the heat supplied from the hot source and the heat absorbed from the cold outdoor.

$$Q_A = Q_H + Q_L \tag{1}$$

For an ideal VM heat pump having ideal regenerators and ideal gas as working fluid, it is directly observed that $Q_H = W_H$ and $Q_L = W_L$ by considering the hot working space and the cold working space as control volumes, respectively. In a real system, however, $Q_H < W_H$ and $Q_L < W_L$ due to the real gas behavior of helium, even if the effective-

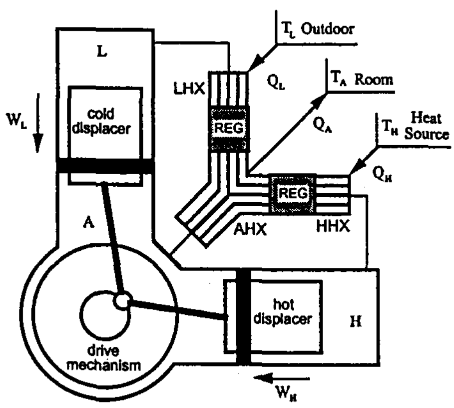

Fig. 1 Configuraion of VM heat pump

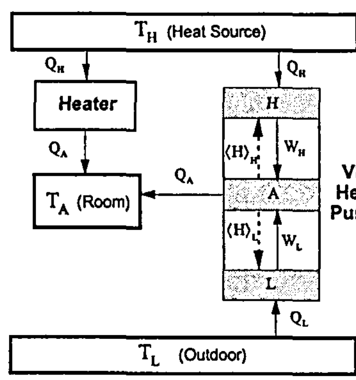

Fig. 2 Energy flow in simple heater and VM heat pump

ness of the regenerators may be 10%.

To quantify the thermal loss due to the non-ideal gas behavior of helium, the energy balance equation can be written for the control volume including the warm working space and the warm heat exchanger as

$$Q_A = W_A - \langle H \rangle = \oint P \cdot dV_A - \oint \dot{m} h \cdot dt \qquad (2)$$

The second term on the right-hand side of Eq. (2) is the net enthalpy flow from the control volume to the surroundings for a cycle. And the work of the warm working space is the sum of the works of the hot and the cold spaces. So the heating capacity of the heat pump per cycle is following.

$$Q_A = W_H + W_L - \oint \dot{m} h \cdot dt \qquad (3)$$

If the working fluid behaves like an ideal gas, the third term of the right-hand side of Eq.(3) vanishes since the gas temperature at the heat exchangers does not vary over a cycle and the enthalpy of an ideal gas is a function of temperature only. The enthalpy of real gas is a function of pressure as well, so the third term may not equal zero. Figure 3 shows the values of the enthalpy departure as a function of pressure for various real gases at 300K. As pressure increases, the enthalpy decreases for the most of gases but increase for helium. The peculiar behavior of helum can be explained by the fact that the critical pressure of helium is so low(T_c =0.227MPa). Generally speaking, at much higher pressures than the critical pressure, the intermolecular force is repulsive rather than attractive and the compressibility factor($Z=P_V/RT$) of helium is greater than 1. Since the opearting pressure of helium in typical VM heat pumps is as high as 1 to 20MPa, the non-ideal gas behavior results in positive value of the net enthalpy flow in Eq.(3) or thermal loss in heating. As far as the present authors are aware, the cycle analysis with the real gas properties of helium in VM heat pump has not been reported. Recently, the authors presented a simple and useful method to modify the conventional adiabatic analysis[5]~[8] in Stirling cycle by incorporating real gas properties of helium[9].

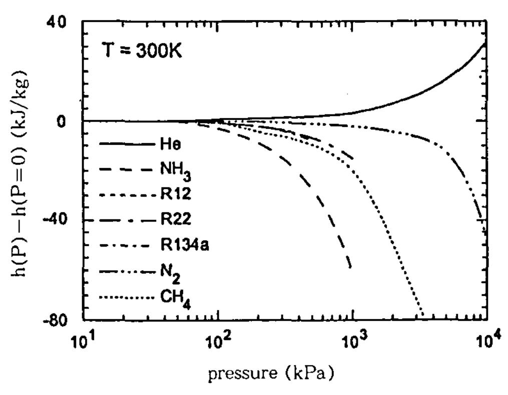

Fig. 3 Enthalpy departure of various gases as a function of pressure at T=300K

This study has been proposed to evaluate the amount of the thermal loss due to the non-ideal gas behavior of helium. The loss is numerically calculated by the conventional adiabatic analysis program with the real gas property of helium for various operating conditions.

2. Analysis Model and Numerical Method

In order to identify the loss due to the non-ideal gas behavior, the effectiveness of every heat exchanger and regenerator is assumed to be 1 in this study. The pressure drop in the gas flow and the heat transfer at the working spaces are neglected. With these simplifying assumptions and the same method of Baik and Chang[9], the mass balance and the energy balance equations are combined to yield the expression for the change of mass in working volumes as

$$dm=\frac{m\left(\beta-\frac{\alpha^2 vT}{C_p}\right)dp+\frac{dV}{v}}{1+\frac{\alpha}{C_P}(h^*-h)} \tag{4}$$

where α, β and C_p are the volume expansion coefficient, the isothermal compressibility, and the specific heat at constant pressure, respectively, and h* denotes the specific enthalpy at the inlet or exit of the working space. In case that the gas flow into the working space or dm>0, h* should be the enthalpy at the temperature of adjacent heat exchanger, because of the 100% effective heat exchangers. In case that the gas flows out or dm<0, h* is equal to h, the enthalpy of gas in the working space, so that Eq.(4) may be simplified as

$$dm=m\left(\beta-\frac{\alpha^2 vT}{C_p}\right)dp+\frac{dV}{v} \tag{5}$$

The change of mass in dead spaces including the three heat exchangers and the two regenerators is determined by the change of pressure as

$$dm_D=\left(\sum_{HX}\frac{\beta V}{v}+\sum_{REG}\int\frac{\beta}{v}dV\right)\cdot dP\equiv A_D\cdot dP \tag{6}$$

since the gas temperature does not vary over time at any location. In Eq.(6), the temperature of gas in regenerator is not spatially uniform and the intergration over the volume is necessary. It should be noted that no numerical differentiation is included in Eq.(4) or (6), since the thermodynamic properties, α, β and C_p, are given by standard compute code as functions of temperature and pressure.

When the changes of mass in the three working spaces and in five dead spaces are expressed in terms of the change of pressure, the sum of the changes can be set to zero since the total mass in the closed system is constant. The equation is rearranged to express the change of pressure as

$$dp=-\frac{\sum_i\left[\dfrac{\frac{dV}{v}}{1+\frac{\alpha}{C_p}(h^*-h)}\right]_i}{A_D+\sum_i\left[\dfrac{m\left(\beta-\frac{\alpha^2 vT}{C_p}\right)}{1+\frac{\alpha}{C_p}(h^*-h)}\right]_i} \tag{7}$$

where the summations should be made for the three working volumes.

Since h* is dependent upon the direction of the flow, numerical integration of Eq.(7) should be made by a trial and error method at every step of integration. While the number of combinations for the flow directions at the three working volumes is eight, only six different cases are possible in practice. First, the directions of the flow in the three working volumes are assumed. Then Eq.(7) is integrated to calculate the change of pressure. The changes of mass in the volumes are calculated with Eq.(4) and the sings of the changes should be matched with the assumed directions of the flows. This integration is performed over a cycle until a periodical steady state is reched.

The integration of the differential equations is performed by the fourth-order Runge-Kutta method. The program is written in FORTRAN 77 and linked with a commercial code[10] of the thermodynamic properties. The iteration is repeated until the relative differences between the initial values and the final values in the pressure and the masses in the working spaces are smaller than 10^{-3}. For a reasonably assumed initial condition, the convergence is obtained within three times of cycle iteration in most cases. The same procedures are performed both with real gas properties and with ideal gas properties. By comparing the two results for various input conditions, the analysis method is justified and the thermal loss due to the non-ideal gas behavior is evaluated.

3. Results and Discussion

For a quantitative discussion, a specific VM heat pump is selected and analyzed in this paper. The specifications of the heat pump, which is based on the design of Carlsen et al.[4], are listed in Table 1. The dead volume and the three working volumes are shown as functions of crank angle in Fig.4. The motion of two displacers is assumed to be sinusoidal. The volumes are non-dimensionalized with the total volume, which is constant over a cycle. The temperature distribution in the two regenerators is assumed to be linear along the flow direction. In this paper, the temperatures of the hot, the warm and the cold heat exchangers are 800K, 330K and 270K, respectively, unless specified otherwise.

Table 1 Specifications of VM heat pump for calcuation

	hot	warm	cold
Cylinder			
Bore(cm)	8.0		10.0
Stroke(cm)	4.2		4.2
Swept volume(cm^3)	211		330
Clearance volume(cm^3)	10.6		16.5
Length of displacer(cm)	8.0		8.0
Appendix gap(cm)	0.1		0.1
Heat exchangers			
Internal tube diameter(mm)	4.2	3.18	2.5
Number of tubes	78	192	320
Regenerators			
Inner diameter(mm)	123		110
Thread diameter(mm)	0.04		0.04
Porosity	0.80		0.78

3.1 Real gas properties and ideal gas properties

In order to compare the analysis results with ideal gas properties and with real gas properties, two different criteria can be considered-the same total mass and the same charging pressure. When we charge helium gas into an evacuated VM heat pump at room temperatue, the charged total mass and the charging pressure are directly related. The first one is to compare the real situation with a hypothetical situation by assuming that the charged gas is an ideal gas with the same mass as the real situation. And the second is to compare the real situation with a hypothetical situation by assuming that the charged gas is an ideal gas with the same charging pressure, It can be stated that the choice out of these two criteria depends only on the point of view.

Figure 5 shows the P-V diagrams for the three working volumes. The work per cycle for each working space can be represented by the area of the contour on P-V diagram. For the hot and the cold working spaces, the state of gas pro-

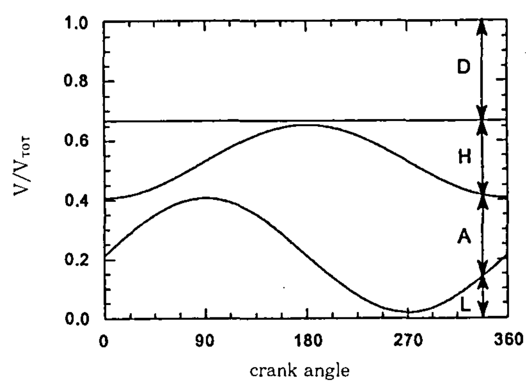

Fig. 4 Dimensionless volume vs crank angle

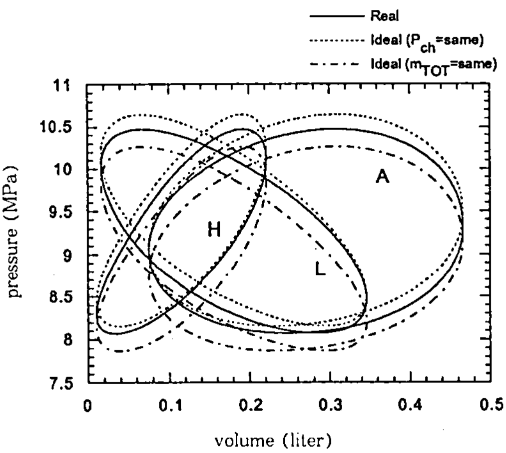

Fig. 5 Pressure-volume diagram for three working spaces (P_{ch}=6.9MPa, m_{tot}=10.2g)

ceeds in the clockwise direction on the diagram and the net work is done by the space. On the contrary, for the warm working space, the state proceeds in the counterclockwise direction and the net work is done to the space. The results with real gas properties are compared with ideal gas properties from two different points of view. The solid curves represent the case of real gas properties when the room-temperature charging pressure is 6.9MPa and the total mass is 10.2g. In the case of ideal gas properties and the same charging pressure, the total mass should be about 10.5g. In the case of ideal gas properties and the same total mass, the room-temperature charging pressure should be about 6.7MPa. It can be noticed that the ideal gas assumption with the same charging pressure results in an overestimation of the operating pressure and the ideal gas assumption with the same total mass results in an underestimation of the operating pressure. As mentioned earlier, these phenomena are explained by the fact that the compressibility factor of helium is greather than 1 at high pressures and at room temperature.

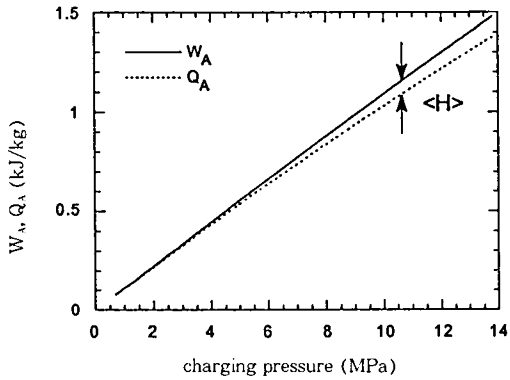

Fig. 6 Q_A, W_A and non-ideal gas loss vs charging pressure

The most significant difference between the cases of real gas and of ideal gas is the relation of the work and the heat at the warm working space. As shown in Eq.(2), in case of ideal gas, the heat to the warm heat exchanger or the heating capacity is identical with the work into the warm working space or the area of the contour in P-V diagram. In case of real gas, however, the heat is smaller than the work and the difference is the loss due to the non-ideal gas behavior of helium.

3.2 Loss due non-ideal gas behavior

For the analysis results with real gas properties, the net enthalpy flow from the warm working space through the two regenerators to the cold and the hot spaces can be calculated by numerical integration over a cycle. The enthalpy flow is considered as the non-ideal gas loss per cycle, since it represents loss of heating capacity.

Figure 6 shows the work into the warm working space and the heating capacity per cycle as functions of the charging pressure at room temperature. As the charging pressure increases, both the work and the heating capacity increase and the difference between them also increases. The difference is the net enthalpy flow from the warm space or the loss in heating due to non-ideal gas behavior. It is observed that the loss is negligibly small when the charging pressure is smaller than 1MPa, but is as much as about 8% of the heating capacity at 10MPa. The loss is the sum of the enthalpy flows to the hot space and to the cold space. Figure 7 shows the composition of two flows as functions of the charging pressure. It is clear that the enthalpy flow to the cold space is much dominant over that to the hot space, since the behavior of real gas departs from that of ideal gas at low temperatures.

The loss due to the non-ideal gas behavior is calculated for various values of input parameters. Figure 8 shows the loss as functions of charging pressure for various temperatures of the hot heat exchanger. The temperature is associated with the kind of heat sources or the combustion conditions of fuel. As the hot temperature increases, the non-ideal gas loss increases, since the amplitude of the pressure oscillation increase for the same charging pressure in the heat pump. Generally speaking, the departure from the ideal gas behaviro decreases as the gas temperatue increases. The effect of the decrease on the non-ideal gas loss is considered to be minor.

Figure 9 shows the loss as functions of charging pressure for various temperatures of the cold heat exchanger. The temperature is associated with the outdoor condition, in case that the heat pump is used as a heater in winter. As the cold temperature increases, the non-ideal gas loss decreases, since both the amplitude of pressure oscillation and the departure from the ideal gas behavior decrease.

The effect of the dead volume ratio on the non-ideal gas loss is shown in Fig.10. The dead volume ratio is defined as the sum of the vol-

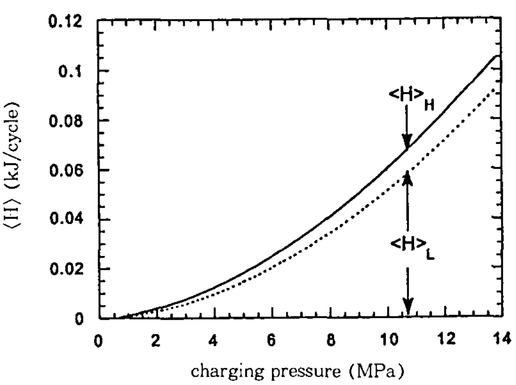

Fig. 7 Non-ideal gas loss VS charging pressure

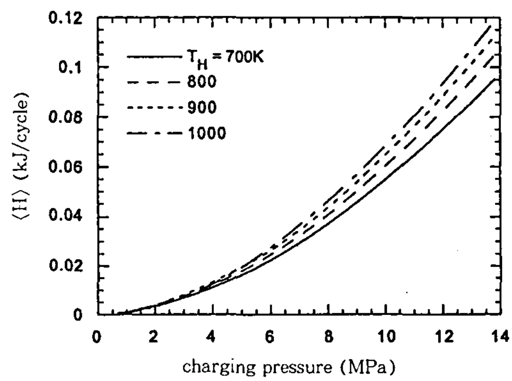

Fig. 8 Non-ideal gas loss VS charging pressure for various T_H

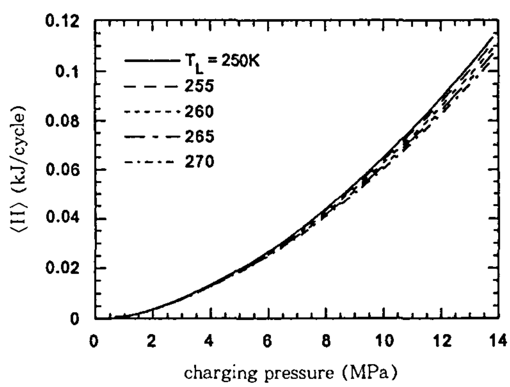

Fig. 9 Non-ideal gas loss VS charging pressure for various T_L

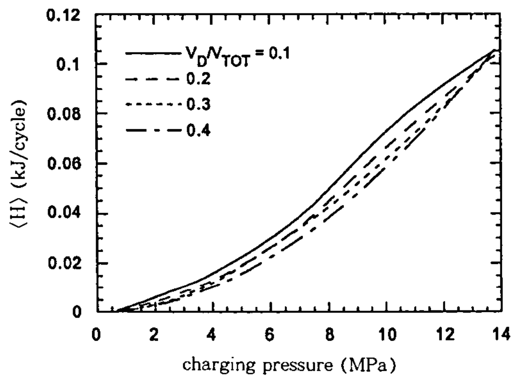

Fig. 10 Non-ideal gas loss VS charging pressure for various dead volume ratios

umes of the three heat exchangers and the two regenerator divided by the total volume. As the dead volume ratio decreases, the amplitude of the pressure oscillation and the non-ideal gas loss increase. On the other hand, it is observed in Fig.10 that the ratio does not affect the amount of the loss at higher charging pressure than 12MPa.

Finally, it is worth mentioning the significane of the loss due to the non-ideal gas behavior, in comparison with other losses in VM heat pump. For VM cryocoolers, this loss is greater than the sum of the heat exchanger loss, the regenerator loss, the shuttle loss, and the conduction loss.[11] The main reason for this is that the cold temperature of the VM cooler is as low as 20K and the behavior of helium is far from that of ideal gas. In the VM heat pumps operated at around room temperature, the loss is not as significant as that in the cryocoolers. In a similar system as in the present study, the heat exchanger loss and the regenerator loss have been reported to be about 20% and 5% of the heating capacity, respectively.[3] It can be clearly concluded that the non-ideal gas loss has to be considered in the analysis and design of VM heat pumps if the operation pressure is as high as 10MPa.

4. Conclusions

An adiabatic analysis program has been developed to calculate the loss due to the non-ideal gas behavior of helium by incorporating the real gas properties. It is noted that when the operating pressure is less than 1MPa, the loss is negligibly small but when the pressure reaches 10MPa, the loss is as much as 8% of the heating capacity. Obviously, the amount of loss is closely related with the enthalpy departure from ideal gas. It is concluded that in VM heat pump analysis, the loss due to non-ideal gas behavior should be included in addition to the typical losses in regenerative machines such as the heat exchanger loss, the pressure drop loss, the reheat loss, or the shuttle loss.

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