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Effect of gap flow on shuttle heat transfer

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Abstract

A comprehensive analysis is performed to investigate the effect of the oscillating flow of gap fluid on the shuttle heat transfer in reciprocating expanders. For a sinusoidal motion of displacer over cylinder having an axial temperature gradient, a new exact expression for shuttle heat transfer is derived from the analytical solution of the velocity and the temperature distributions for the gap fluid and the two walls. Through a rigorous analysis, the most significant parameter in the shuttle phenomena is proven to be the ratio of the inertial force to the viscous force in the oscillating fluid. For the ratio values smaller than unity, the predicted shuttle heat transfer from the present expression is in good agreement with the previously published results. For large ratios, however, a notable discrepancy exists between the results of the present analysis and the previous investigations. The reason for this discrepancy is that the wall-to-wall heat flux is not in phase with the temperature difference because of the fluid motion that was not included in the previous investigations. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Fluid dynamics; Heat transfer; Expander

1. Introduction

Reciprocating expander is a key component in many cryogenic refrigerators such as Stirling, Gifford–McMahon, Vuilleumier and Claude cycle coolers. When an axial temperature gradient exists along the cylinder wall, the reciprocating motion of the displacer causes an extra heat transfer from the high to the low temperature region in addition to the wall conduction. This motional heat transfer is usually referred to as shuttle heat transfer. Since the heat transfer always results in a loss of refrigeration, it is important to accurately estimate and reduce the amount of shuttle heat transfer.

The basic principle of the shuttle heat transfer is explained in Fig. 1. The dot at the center of the displacer indicates its relative axial position with respect to the cylinder. While the displacer is moving towards the high temperature side from the center of the stroke, the displacer wall has a lower temperature than the cylinder wall of the same axial position so that heat is transferred from the cylinder to the displacer. Similarly, while the displacer is moving towards the low temperature side, the displacer wall has a higher temperature and heat is transferred from the displacer to the cylinder. As the cycle proceeds, the displacer repeatedly receives heat from the higher temperature region and rejects heat to the lower temperature region, which creates a net enthalpy flow from the high to the low temperature direction. In the periodic steady state, the cyclic average of the enthalpy flow at a fixed axial position is the shuttle heat transfer.

Several mathematical expressions have been introduced to date to estimate the amount of the shuttle heat transfer. Zimmerman and Longthworth [1] derived an expression based upon the assumptions that the walls of the displacer and the cylinder have an infinite heat capacity and that the displacer has a square-wave motion. Rios [2] presented an approximate solution by linearization and application of Fourier series. Radebaugh and Zimmerman [3] developed an approximate solution by heat conduction analysis of a semi-infinite plate with a sinusoidal surface temperature distribution. In their analysis, the thermal resistance of gas in the gap between the walls was initially neglected in the calculation of the wall temperatures, and was included later by an approximate method. Several years ago, Nishio et al. [4] reported an approximate solution by including the

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Nomenclature

c	specific heat
D	outer diameter of displacer
k	thermal conductivity
L	length of displacer
Pr	Prandtl number, ν/α
Q	shuttle heat transfer
S	stroke of displacer
t	time
T	temperature
u	axial velocity of gap fluid
x	radial distance from surface
z	axial coordinate in stationary reference frame
Z	axial coordinate in moving reference frame

Greek letters

α	thermal diffusivity, $k/\rho c$
δ	gap clearance between displacer and cylinder
ν	kinematic viscosity
θ	complex temperature
ρ	density
ω	angular velocity of oscillation
Ψ	complex constant

Subscripts

1	displacer wall
2	cylinder wall

temperature oscillations for both of the displacer and the cylinder. In their work, a thermal resistance model was employed and the analytical results were compared with the corresponding numerical simulation results. Baik and Chang published an exact mathematical solution [5] for the shuttle heat transfer by including the wall properties of both the displacer and cylinder, and also presented in a simple and useful form of the expression [6].

In spite of the series of progress, the previous estimations of the shuttle heat transfer may not be completely reliable, mainly because they are all based upon the assumption that the wall-to-wall heat transfer is directly proportional to the difference between the two surface temperatures. Since the conventional concept of the convection heat transfer is developed for a steady flow of fluid, the assumption may be true for relatively low speed machines. In cases involving fast oscillations, however, the wall-to-wall heat transfer may not be in phase with the temperature difference, depending upon the rate of momentum and heat diffusion across the viscous fluid in the gap. Recently, Naka et al. [7] included the gap fluid motion and the end effect of a displacer in their numerical analysis using a commercial software. Their result was in good agreement with the existing results, since they dealt with only a low frequency case.

The proposed analysis attempts to investigate the effect of the oscillating flow of the gap fluid and obtain a more accurate estimate of the shuttle heat transfer. To achieve this goal, exact mathematical solutions for the velocity and temperature distributions in the two solid walls and the gap fluid are obtained, and a functional expression for the shuttle heat transfer is derived from

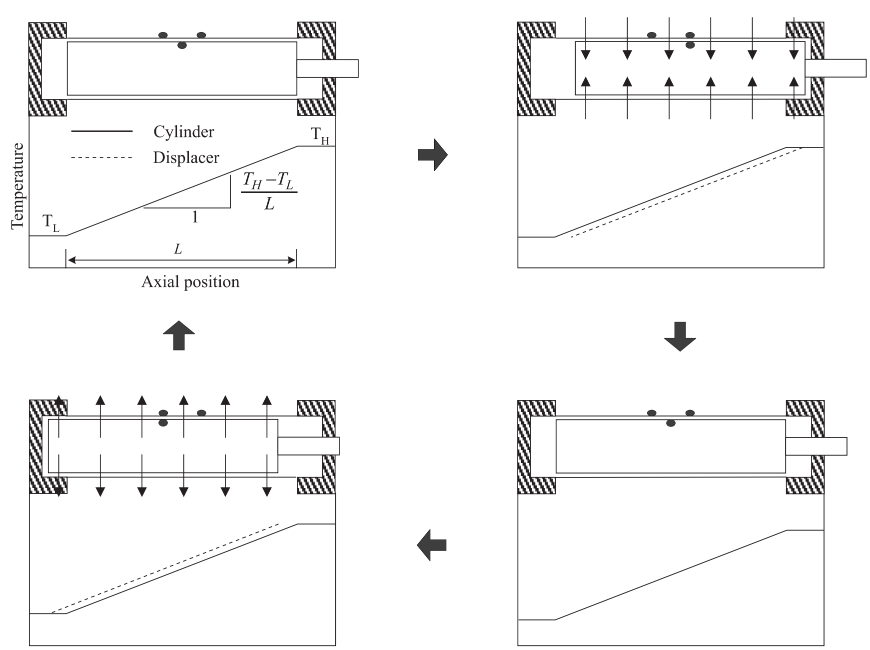

Fig. 1. Basic principle of shuttle heat transfer.

the exact solutions. The usefulness of the expression is evaluated and its physical interpretation is presented.

2. Analysis model

As shown in Fig. 2, the displacer reciprocates over the cylinder having an axial temperature gradient with an angular speed ω and a stroke S. The stationary axial coordinate z is measured from the center of the cylinder and the moving axial coordinate Z is measured from the center of the displacer. The radial coordinates, x_1, x and x_2, are measured from the surface of the cylinder or the displacer. The subscripts 1 and 2 denote the displacer and the cylinder, respectively, and the variables without subscripts represent the gap fluid.

For simplicity, the following assumptions are made in this analysis:

1. The system is in a cyclic steady state with a sinusoidal motion of the displacer. Therefore the two axial coordinates have the following relation:

$$z = Z + \frac{S}{2}\cos\omega t. \tag{1}$$

2. The axial temperature gradient of the two walls and the fluid in the gap is a constant, $(T_H - T_L)/L$. This condition holds when the material properties do not vary.
3. The wall thickness of the displacer or the cylinder is much smaller than the diameter, but greater than the thermal penetration depth so that the displacer and the cylinder can be considered as two semi-infinite flat plates. The thermal penetration depth was verified for typical conditions by Baik and Chang [5].
4. The gap fluid is Newtonian and oscillated only by the motion of displacer. The pressure gradient, the compressibility of fluid, and the end effect of flow are neglected.
5. The radiation heat transfer is negligible.

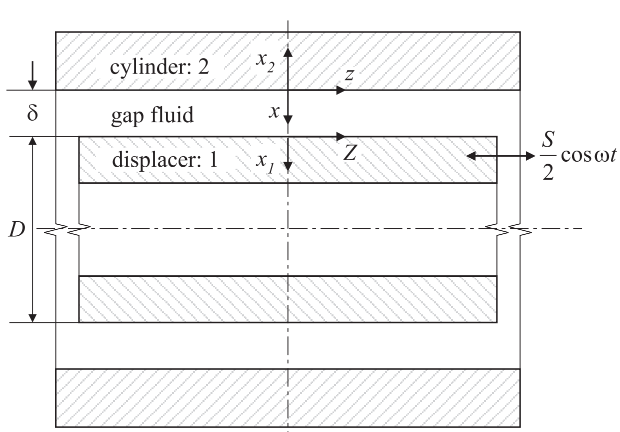

Fig. 2. Coordinates of displacer–cylinder system.

These assumptions can be justified in most practical situations where the shuttle heat transfer is significant. It should be noted that neither an infinite heat capacity of the displacer or cylinder nor a constant heat transfer coefficient between two surfaces has been assumed in this analysis.

3. Velocity distribution of gap fluid

The velocity of the gap fluid can be determined by the Navier–Stokes equation for the flow between two parallel plates, one of which oscillates sinusoidally, in accordance with assumptions 1–4. Since the axial velocity u is independent of the axial position and the radial velocity is zero, the equation can be reduced [8,9] to

$$\frac{\partial u(t,x)}{\partial t} = \nu \frac{\partial^2 u(t,x)}{\partial x^2} \tag{2}$$

subject to the non-slip boundary conditions

$$u(t,0) = 0, \qquad u(t,\delta) = -\frac{\omega S}{2}\sin\omega t. \tag{3}$$

This equation can be readily solved by the method of complex variables, in the same way as the Stokes' second problem [8,9]:

$$u(t,x) = \mathrm{Re}\left\{ \frac{\mathrm{i}\omega S}{2} \frac{\sinh\left[(1+\mathrm{i})(\sqrt{\omega/2\nu})x\right]}{\sinh\left[(1+\mathrm{i})(\sqrt{\omega/2\nu})\delta\right]} \mathrm{e}^{\mathrm{i}\omega t} \right\}, \tag{4}$$

where $\mathrm{i} = \sqrt{-1}$ and Re denotes the real part of a complex variable. The correctness of Eq. (4) can be verified by substituting into the governing equation and the boundary conditions.

It is worthwhile to discuss at this point about the significance of Eq. (4) in practice. Fig. 3 shows the axial velocity profiles at π/6 increment over a half period of down stroke for two different frequencies of $\omega/2\pi = 0.5$ and 20 Hz. The gap is set at δ =0.7 mm [7], and the gap fluid is helium at 200 K and 10 atm. The two frequencies represent approximately the lower and the upper limits in practical cryocoolers. For 0.5 Hz, the oscillation is so slow that the viscous force of the fluid is dominant over the inertial force. Therefore the velocity profile is almost linear as in the steady-state solution and the shear stress distribuation in fluid is constant across the gap. On the contrary, for 20 Hz, the inertial force is significant, and the fluid near the center of the gap may flow in the opposite direction of the displacer. It is clear form Eq. (4) that this behavior can be generalized by a dimensionless variable, $\delta\sqrt{\omega/\nu}$, which can be defined as the ratio of the inertial to the viscous force in oscillating flows [10,11].

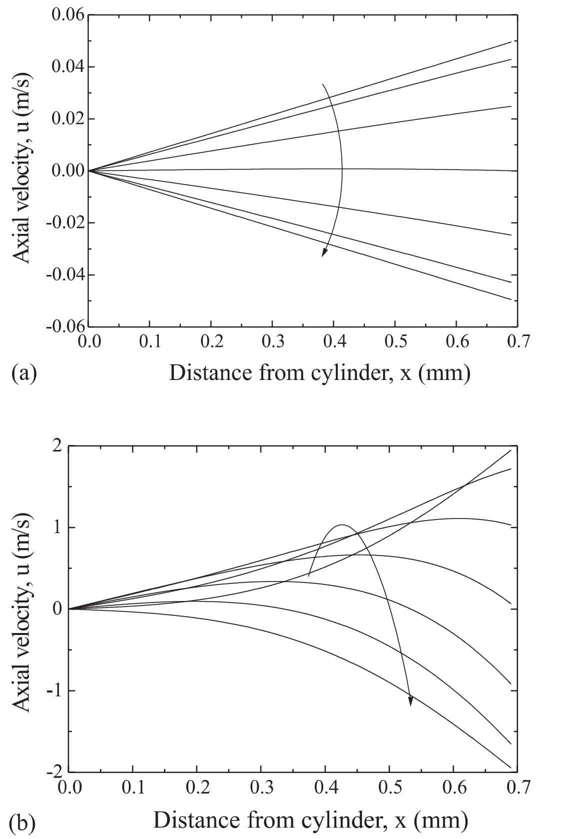

Fig. 3. Axial velocity profiles in $\pi/6$ increments for half period: (a) $\omega/2\pi = 0.5$ Hz, (b) $\omega/2\pi = 20$ Hz.

4. Temperature distribution

The governing equations for the temperature of the displacer wall, the gap fluid and the cylinder wall are

$$\frac{\partial T_1(t,x_1,Z)}{\partial t} = \alpha_1 \frac{\partial^2 T_1(t,x_1,Z)}{\partial x_1^2}, \tag{5}$$

$$\frac{\partial T(t,x,z)}{\partial t} + u(t,x)\frac{\partial T(t,x,z)}{\partial z} = \alpha \frac{\partial^2 T(t,x,z)}{\partial x^2}, \tag{6}$$

$$\frac{\partial T_2(t,x_2,z)}{\partial t} = \alpha_2 \frac{\partial^2 T_2(t,x_2,z)}{\partial x_2^2}, \tag{7}$$

respectively. The axial coordinate, Z, in Eq. (5) is moving with the displacer, while the axial coordinate, z, in Eqs. (6) and (7) is stationary. In the second term of Eq. (6), the axial velocity of the gap fluid, $u(t,x)$ is given by Eq. (4) and the axial temperature gradient is a constant according to assumption 2 even though the temperature may be a function of time and space.

Two boundary conditions, according to assumptions (2) and (3), are

$$T_1(t,\infty,Z) = T_0 + \frac{T_H - T_L}{L} Z, \tag{8}$$

$$T_2(t,\infty,z) = T_0 + \frac{T_H - T_L}{L} z, \tag{9}$$

where T_0 is a reference temperature, which is simply selected as the cylinder temperature at a location with $z = 0$ and radially far apart from the surface. It is noted that the two temperatures from Eqs. (8) and (9) are identical when $z = Z$ or the displacer is at the center of the stroke. Four other boundary conditions are obtained from the continuity of temperature and heat flux at the surfaces:

$$T_1\left(t, 0, z - \frac{S}{2}\cos\omega t\right) = T(t,\delta,z), \tag{10}$$

$$k_1 \frac{\partial T_1(t,0,z-\frac{S}{2}\cos\omega t)}{\partial x_1} = k\frac{\partial T(t,\delta,z)}{\partial x}, \tag{11}$$

$$T_2(t,0,z) = T(t,0,z), \tag{12}$$

$$k_2 \frac{\partial T_2(t,0,z)}{\partial x_2} = -k\frac{\partial T(t,0,z)}{\partial x}, \tag{13}$$

where k_1, k, k_2 are the thermal conductivities and Z has been replaced with a function of z and t via Eq. (1). The initial conditions are not necessary in a cyclic steady state.

The temperature for the two walls and the gap fluid is found by solving Eqs. (5)–(7) simultaneously with the boundary conditions (8)–(13), by the method of complex variables. In this analysis, complex temperatures, θ_1, θ, θ_2 are defined [12,13] as

$$T_1 = \mathrm{Re}(\theta_1), \quad T = \mathrm{Re}(\theta), \quad T_2 = \mathrm{Re}(\theta_2). \tag{14}$$

Eqs. (5)–(13) can be expressed in the complex forms by replacing Ts with θs, and $\cos\omega t$ with $e^{i\omega t}$. In the second term of Eq. (6), $u(t,x)$ can be simply replaced with the complex velocity of Eq. (4), since the axial temperature gradient is a constant.

If the three unknown complex temperatures are assumed to be

$$\theta_1(t,x_1,z) = T_0 + \frac{T_H - T_L}{L}\left(z - \frac{S}{2}e^{i\omega t}\right) + \theta_{10}(x_1)e^{i\omega t}, \tag{15}$$

$$\theta(t,x,z) = T_0 + \frac{T_H - T_L}{L} z + \theta_0(x)e^{i\omega t}, \tag{16}$$

$$\theta_2(t,x_2,z) = T_0 + \frac{T_H - T_L}{L} z + \theta_{20}(x_2)e^{i\omega t} \tag{17}$$

and substituted into the governing equations, the $e^{i\omega t}$ factors drop out and the partial differential equations are reduced to ordinary differential equations for $\theta_{10}(x_1)$, $\theta_0(x)$ and $\theta_{20}(x_2)$. A slightly lengthy but straightforward procedure leads to the exact solution for $\theta_{10}(x_1)$. When Pr is not unity

$$\theta_{10}(x_1) = \frac{S}{2}\frac{T_H - T_L}{L}\frac{1-\Psi}{1-Pr} e^{-(1+i)\sqrt{\frac{\omega}{2\alpha_1}}x_1}, \tag{18}$$

where Ψ is a complex constant defined as

$$\Psi = \left(\left\{\sigma_2 \sinh\left[(1+\mathrm{i})\sqrt{\frac{\omega}{2\alpha}}\delta\right] + \cosh\left[(1+\mathrm{i})\sqrt{\frac{\omega}{2\alpha}}\delta\right]\right\} \times\left\{\sigma_1 \sinh\left[(1+\mathrm{i})\sqrt{\frac{\omega}{2v}}\delta\right] + \sqrt{Pr}\cosh\left[(1+\mathrm{i})\sqrt{\frac{\omega}{2v}}\delta\right]\right\} - \sqrt{Pr}\right)\Big/\left(\sinh\left[(1+\mathrm{i})\sqrt{\frac{\omega}{2v}}\delta\right]\left\{(1+\sigma_1\sigma_2) \times \sinh\left[(1+\mathrm{i})\sqrt{\frac{\omega}{2\alpha}}\delta\right] + (\sigma_1+\sigma_2) \times \cosh\left[(1+\mathrm{i})\sqrt{\frac{\omega}{2\alpha}}\delta\right]\right\}\right). \quad (19)$$

In Eqs. (18) and (19), three dimensionless parameters,

$$Pr = \frac{v}{\alpha}, \quad \sigma_1 = \sqrt{\frac{k_1\rho_1 c_1}{k\rho c}}, \quad \sigma_2 = \sqrt{\frac{k_2\rho_2 c_2}{k\rho c}} \quad (20)$$

represent the thermophysical properties of the gap fluid, the displacer and the cylinder walls. It should be noted that Ψ is a dimensionless complex function expressed in terms of only four real parameters, Pr, σ_1, σ_2 and $\delta\sqrt{\omega/v}$, since $\delta\sqrt{\omega/\alpha} = \delta\sqrt{\omega/v}\sqrt{Pr}$.

The procedure to verify that the solution satisfies the set of simultaneous equations can be quite tedious. However, it is simple to prove that Eq. (15) with Eq. (18) satisfies Eqs. (5) and (8), since the real part of the exponent in Eq. (18) is negative. In the special case that $Pr = 1$, the solution can be obtained by a method known as the variation of parameters, which is not presented in this paper because it is not important in practice.

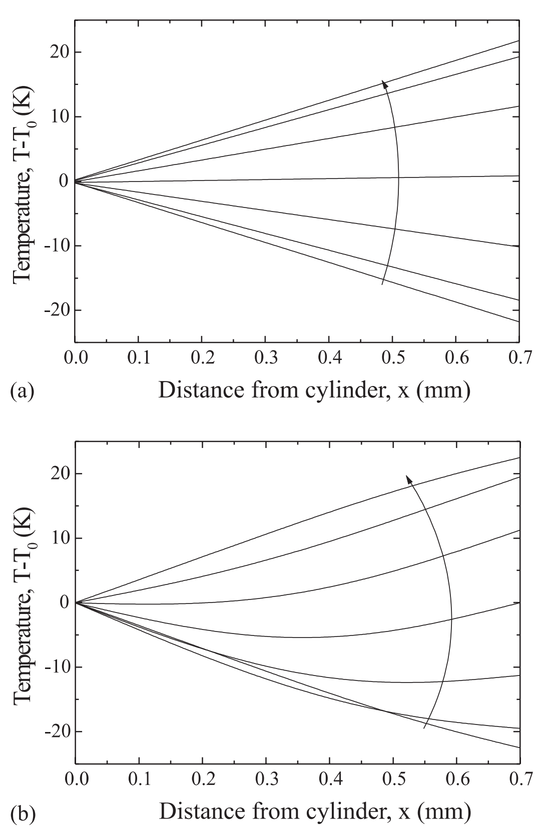

Fig. 4. Temperature profiles in $\pi/6$ increments for half period: (a) $\omega/2\pi$ =0.5 Hz, (b) $\omega/2\pi$ =20 Hz.

Table 1
Specifications of shuttle heat transfer system in the sample calculation

Displacer	Material	–	Bakelite
	Outer diameter	D	60 mm
	Stroke	S	32 mm
Cylinder	Material	–	Stainless steel
	Axial temperature gradient	$(T_H - T_L)/L$	1.43 K/mm
Gap	Gas	–	Helium
	Gap clearance	δ	0.7 mm
	Average temperature	T_0	200 K
	Average pressure	–	10 atm

As described in the following section, the shuttle heat transfer is derived from the temperature distribution of the displacer wall. The mathematical expressions for the temperature distributions of the fluid and the cylinder wall will not be presented here due to a space limitation. Instead, the temperature profiles in the gap fluid are graphically shown in Fig. 4 for a half period of down stroke for two frequencies of $\omega/2\pi = 0.5$ and 20 Hz. The conditions for this calculation are given in Table 1. During the half stroke toward the negative z direction, the temperature increases for most regions as shown in Fig. 1. The temperature variation at cylinder surface ($x = 0$) is very small due to its large heat capacity. For 0.5 Hz, the temperature profile is almost linear as in a steady-state solution and the oscillation of the temperature difference between the two surfaces (at $x = 0$ and δ) is in phase with that of the heat flux at the surfaces. For 20 Hz, however, the temperature profile is not linear and the temperature difference is not proportional to the heat flux at surfaces. It can be stated that the heat flux at the displacer surface (at $x = \delta$) always leads that at the cylincer surface(at $x = 0$) in the phase angle.

5. Shuttle heat transfer

Once the temperature of the displacer wall is obtained, the shuttle heat transfer can now be calculated. At an arbitrary axial position, the shuttle heat transfer can be defined as the net enthalpy flow rate from the high to the low temperature side, or in the negative z direction. A cycle-averaged net enthalpy flow rate of the displacer wall can be obtained by integrating over a period and the displacer wall area.

$$Q = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \times \int_0^{\infty} \rho_1 c_1 T_1(t, x_1, z)\left(\frac{\omega S}{2}\sin\omega t\right)\pi D\,\mathrm{d}x_1\,\mathrm{d}t, \quad (21)$$

where the parentheses denotes the velocity of the displacer in the negative z direction as given by Eq. (3) and $\pi D dx_1$ denotes the infinitesimal cross-sectional area of the displacer wall according to assumption (3). Since the heat capacity of the gap fluid in most practical cases is negligibly small, its contribution to the shuttle heat transfer is omitted in Eq. (21). The result of integrating Eq. (21) is

$$Q = \frac{T_H - T_L}{L}\frac{\pi D S^2}{8}k_1\sqrt{\frac{\omega}{2\alpha_1}}\frac{\mathrm{Re}(1-\Psi) - \mathrm{Im}(1-\Psi)}{1 - Pr}, \qquad (22)$$

where Re and Im denote the real and the imaginary parts of the complex number, respectively, and Ψ is given by Eq. (19). It is noted that Q is independent of z, since the axial temperature gradient is a constant and the end effect is neglected.

According to Eq. (22), the shuttle heat transfer is proportional to the axial temperature gradient, the diameter of the displacer, and the square of the stroke. On the other hand, the dependence on the oscillating frequency, the gap clearance between the two walls, and the properties of the fluid or the walls is rather complicated. It is interesting to note that the wall thickness of the displacer or the cylinder is crucial in the axial wall conduction, but does not affect the shuttle heat transfer given by Eq. (22), as long as the thickness is greater than the thermal penetration depth according to assumption 3.

6. Results and discussion

The validity of the new expression for the shuttle heat transfer, Eq. (22), can be best confirmed by an experiment. Unfortunately, however, the shuttle loss cannot be separately measured in a cryocooler and no experimental data are available for comparison. On the other hand, for the purpose of a quantitative evaluation of the proposed analysis, the new expression can be compared with the existing analytical expressions in a specific situation. The materials and the dimensions for this comparison are given in Table 1. Most of the contents in Table 1 are taken from Naka et. al. [7], because they have presented only a few results of numerical simulation while other analytical expressions can produce numerical data in any case.

Fig. 5 plots the shuttle heat transfer as a function of oscillating frequency from the previously published results and the proposed expression, Eq. (22). For Radebaugh and Zimmerman [3], Nishio et. al. [4] and Chang and Baik [6], the wall-to-wall heat transfer has been assumed to be proportional to the temperature difference of the two surfaces. In the coordinate system of Fig. 2

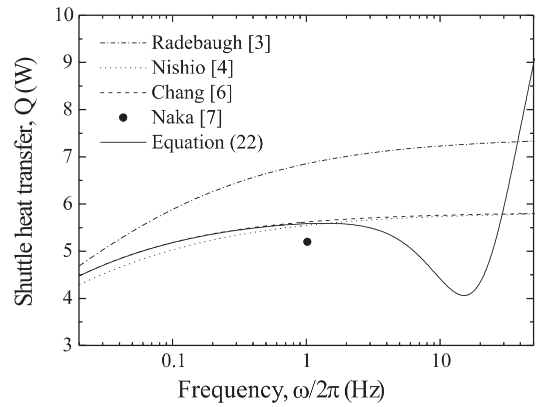

Fig. 5. Comparison of the current and existing expressions for shuttle heat transfer as a function of oscillating frequency.

$$-k_1\frac{\partial T_1\left(t,0,z-\frac{S}{2}\cos\omega t\right)}{\partial x_1} = h\left[T_2(t,0,z) - T_1\left(t,0,z-\frac{S}{2}\cos\omega t\right)\right] = k_2\frac{\partial T_2(t,0,z)}{\partial x_2}, \qquad (23)$$

where h is the convection heat transfer coefficient that has a constant value

$$h = \frac{k}{\delta}. \qquad (24)$$

Radebaugh and Zimmerman [3] have over-estimated the shuttle heat transfer at high frequencies, as they have assumed an infinite heat capacity hence no temperature oscillation of the cylinder wall. A fairly good agreement is observed between the results of Nishio et al. [4] and Chang and Baik. [6] The minor difference is due to the assumption imposed by Nishio et al. that the phase shift of the surface temperature oscillation is negligible in their approximate thermal resistance model. The numerical result from Naka et. al. [7] indicates a slightly lower value at 1 Hz, as they have included the axial end effect of the displacer motion and assumed that the helium gas is at the atmospheric pressure. Owing to exactness and simplicity, the expression from Chang and Baik [6]

$$Q = \frac{T_H - T_L}{L}\frac{\pi D S^2}{8} \times \left(\frac{1}{k_1}\sqrt{\frac{\alpha_1}{2\omega}} + \frac{1}{k_2}\sqrt{\frac{\alpha_2}{2\omega}} + \frac{\delta}{k}\right) \Bigg/ \left(\left(\frac{1}{k_1}\sqrt{\frac{\alpha_1}{2\omega}} + \frac{1}{k_2}\sqrt{\frac{\alpha_2}{2\omega}}\right)^2 + \left(\frac{1}{k_1}\sqrt{\frac{\alpha_1}{2\omega}} + \frac{1}{k_2}\sqrt{\frac{\alpha_2}{2\omega}} + \frac{\delta}{k}\right)^2\right) \qquad (25)$$

has been the most useful one so far as Eqs. (23) and (24) hold true. It should be recognized that Eq. (22) contains only two more variables, ν and ρc of the fluid, than Eq. (25), noting that $Pr = \nu/\alpha$ and $\alpha = k/\rho c$. Obviously, the kinematic viscosity, ν, and the heat capacity, ρc, are critical in determining the velocity and the temperature profiles of the gap fluid, as illustrated in Figs. 3 and 4, respectively.

The shuttle heat transfer from Eq. (22) shows an accurate agreement with Eq. (25) at frequencies lower than 1 Hz. As the frequency increases, however, it decreases to a considerably lower value and then increases again to higher values compared to the existing results. This seemingly unusual behavior results from the obvious fact that Eq. (23) is not true any more at higher frequencies. It should be emphasized again that the new expression is based upon the exact solution satisfying the energy equations and the boundary conditions and that the data have been generated from a single mathematical expression that coincides with Chang et al. [6] at low frequencies.

As demonstrated in Fig. 4, the surface heat flux is not in phase with the temperature difference at relatively high frequencies. Therefore, Eq. (23) cannot be used irrespective of the value of the heat transfer coefficient h. In order to illustrate how the phase shift affects the shuttle heat transfer, the locus of the complex temperature at the displacer surface, $\theta_{10}(0)$, is plotted on the complex plane of Fig. 6 as the frequency is varied. As noticed in Eqs. (18) and (22), the shuttle heat transfer is proportional to the difference of the real and imaginary parts of the complex temperature so that the straight lines indicate the points of the same shuttle heat transfer. As the frequency increases, the complex temperature asymptotically approaches the point marked by a hollow circle in which the phase angle is $-\pi/4$ and the shuttle heat transfer is approximately 5.8 W, as given by Chang and Baik [6]. The exact complex number from Eq. (18) coincides with Chang and Baik for up to 1 Hz, but makes a sharp clockwise turn and runs away as the frequency is farther increased. The reason for this behavior is that the phase angle of the temperature oscillation at the displacer surface is significantly influenced by the gap flow at high frequencies as the inertial force of fluid becomes dominant over the viscous force.

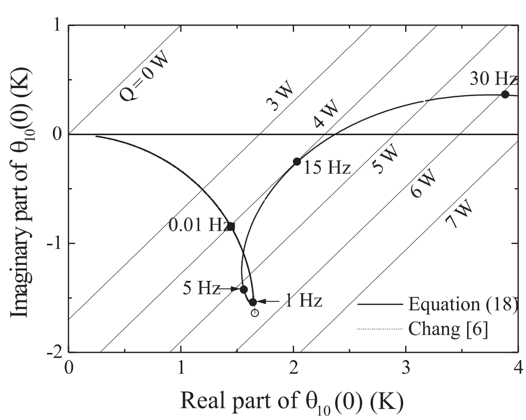

Fig. 6. Locus of complex temperature at displacer surface, $\theta_{10}(0)$, on complex plane at various frequencies.

In practice, the key point of the new expression is that the shuttle heat transfer is not a monotonically increasing function of the oscillating frequency, but has a local minimum at about 15 Hz in this case. The result can be generalized by employing the method of dimensional analysis. As discussed in Eq. (22), the shuttle heat transfer is associated with four dimensionless variables, Pr, σ_1, σ_2 and $\delta\sqrt{\omega/v}$, in addition to the explicit variables such as the axial temperature gradient, the diameter or stroke of the displacer. Since the first three dimensionless variables are determined by the fluid or the wall materials, the shuttle heat transfer can be minimized by designing an appropriate value of $\delta\sqrt{\omega/v}$. It may well be predicted for the typical combination of helium (gap fluid), bakelite (displacer) and stainless steel (cylinder) that the optimum value for $\delta\sqrt{\omega/v}$ is approximately 2.7.

7. Conclusions

An analytical expression is presented for the shuttle heat transfer by taking into account the oscillating flow of the gap fluid. It can be concluded from the exact expression that the gap flow does affect the shuttle phenomena when the inertial force in the oscillating flow is relatively significant over the viscous force. In most of practical operating conditions, the predicted shuttle heat transfer will be less than the previously published value, depending upon the ratio of the two forces. The result of this paper is readily applicable to many cryogenic systems in which the shuttle heat transfer is significant.

Acknowledgements

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