Optimal Integration of Binary Current Lead and Cryocooler

H.M. Chang[1] and S.W. Van Sciver[2]

[1]Hong Ik University, Dept. of Mechanical Engineering
Seoul, 121-791   Korea

[2]National High Magnetic Field Lab
Tallahassee, FL   32306

ABSTRACT

An optimal integration of a binary current lead and a two-stage cryocooler has been analytically sought to minimize the required refrigerator power. The binary current lead is a series combination of a normal metal conductor at the warmer part and a high Tc superconductor (HTS) at the colder part. The lead is cooled by direct contacts with the first stage of the two-stage cryocooler at the joint and with the second stage at the cold end. No helium boil-off gas is present.

A new and simple analytical method is developed to calculate the cooling loads at the two locations in the binary lead. A mathematical expression for the required power input for the loads is derived by incorporating a model depicting the performance of actual cryocoolers. With a new graphical method, the optimal conditions are found for the cooling temperature at the joint and the dimensions of the two parts to minimize the refrigerator power per unit current.

The results show that there exists an optimal relation between the length and the current density of the metal lead, which is independent of the HTS part or the cryocooler. It is also demonstrated that the current density of the HTS and the joint temperature have unique optimal values respectively to minimize the refrigerator power per unit current, when the length of the HTS part and its critical properties are given. The actual power input to the cryocooler in the optimal conditions is compared with its minimum as a thermodynamic limit, which can be obtained with reversible refrigerators. In addition, a useful dimensionless number is introduced for the optimal cooling of the binary current leads.

INTRODUCTION

A binary current lead is a series combination of normal metal conductor as a higher temperature part and high Tc superconductor (HTS) as a lower temperature part. Since the HTS materials are perfect electrical conductors and have much lower thermal conductivity than the normal metals, the heat leak to the cryogenic temperatures through the binary leads could be considerably smaller than that through the conventional metallic leads. A number of studies[1-6] have been performed during the past several years, to apply the bulk HTS materials to the leads carrying a high current density and to develop the effective cooling technology. The cooling method for the binary leads could be quite different from the standard helium-vapor-cooling of

the conventional metallic lead, depending on how the liquid cryogens and/or the cryocoolers are employed[1].

Recent progress in the development of 4 K Gifford-McMahon refrigerators has raised the possibility of the liquid-free or the cryocooler-cooled superconducting magnets[7,8]. Since there is no liquid cryogen in those superconducting systems, the current leads should be conduction-cooled in vacuum by contact with cryocoolers. Two typical configurations of the superconducting systems cooled by two-stage cryocoolers are schematically shown in Figure 1. The second stage of the two-stage GM cryocooler absorbs heat from the magnet and the cold end of the HTS current lead, while the first stage cools the radiation shield and the joint between the two parts of the binary lead. The cooling at the joint is necessary to reduce the amount of heat leak to the lower temperatures and to maintain the HTS in a superconducting state. The present authors think that the feasibility of the conduction-cooled HTS current lead has been demonstrated by the recent construction and operation of several prototypes[7,8] and the next crucial step towards the practical application could be the development of energy-efficient current leads.

The cooling of the HTS leads without the boil-off helium gas has been partly considered in some of the previous publications[1-6]. Most of this research work is, however, related with the design or the analysis for the HTS leads whose ends were cooled by liquid nitrogen or liquid helium and still might not provide enough information on the optimal cooling scheme for the liquid-free HTS leads. For conventional metallic leads, the conduction-cooling method was examined and completely optimized by Hilal[9]. In the theoretical work, Hilal showed by the method of calculus of variations that the refrigerator power could reach an absolute minimum with optimally distributed Carnot refrigerators and optimally sized leads. A few years before Hilal's work, Bejan and Smith[10] derived an absolute minimum of the refrigerator power required to cool a given geometry of mechanical supports for cryogenic apparatus. From a thermodynamic point of view, the mechanical supports are quite similar to the HTS current leads that do not generate heat in a superconducting state.

Recently, the present authors published a new optimization technique for the conduction-cooling of the binary current leads[11], by combining the two optimization methods mentioned above. They have revealed that the refrigerator power has an absolute minimum as a thermodynamic limit, when the lead is cooled by optimally distributed Carnot refrigerator along the length of the lead and the dimensions of the lead are optimized. They have also presented the optimal conditions for the binary current lead cooled by a two-stage Carnot refrigerator. These results are considered as thermodynamic limits, because the refrigerator has been assumed to be reversible and the HTS is marginally superconducting at the optimal conditions.
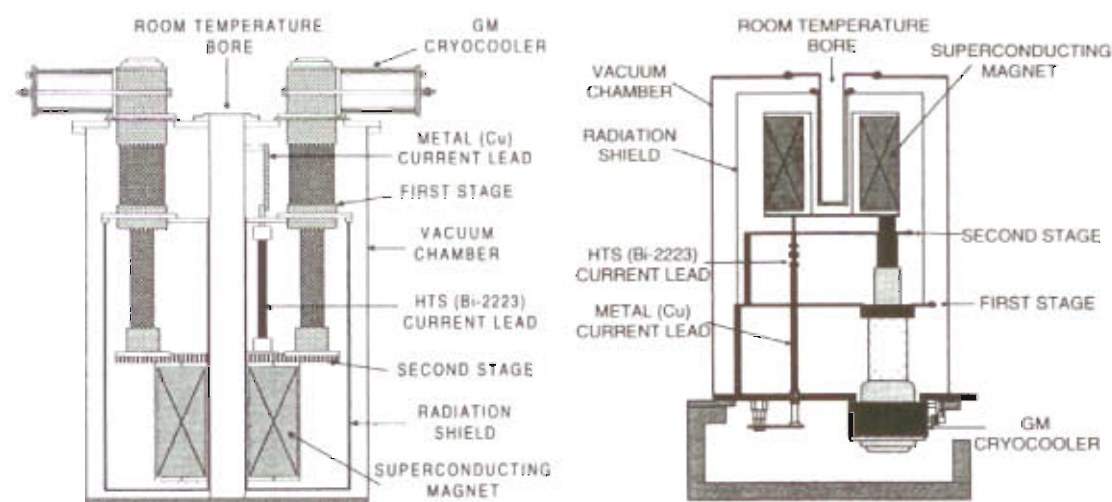

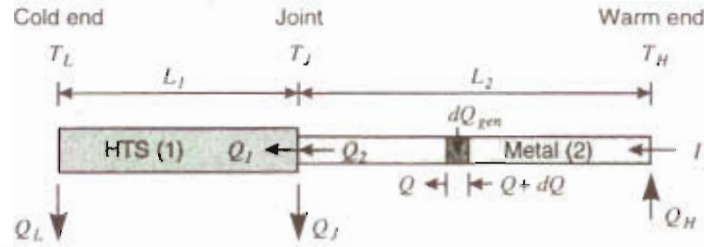**Figure 1.** Typical configurations of cryocooled-cooled superconducting magnet systems .

Figure 2. Binary current lead cooled at cold end and at joint by a two-stage cryocooler.

This paper aims at the optimal operating conditions for the binary current lead cooled by an actual two-stage cryocooler to minimize the required refrigerator power. In order to achieve a practical usefulness, the optimization should include the performance characteristics of the contemporary cryocoolers and the realistic stability margins. A new analytical and graphical method is presented first with specific examples for a quantitative discussion. Then, a more general optimization technique is discussed for its application to every kind of binary lead cooled by a two-stage refrigerator.

COOLING LOADS

A binary current lead cooled by a two-stage cryocooler is schematically shown in Figure 2. The HTS part and the normal metal part of the binary lead are denoted by subscripts 1 and 2, respectively. The heat current through the lead in a direction from the warm to the cold end is defined as Q. The heat removed at the cold end by the second stage of the cryocooler is denoted by Q_L and the heat removed at the joint by the first stage is denoted by Q_J. Q_J is the difference between the heat from the metal at the joint, Q_2, and the heat to the HTS at the joint, Q_1. It is assumed that the HTS does not generate heat in a superconducting state. T_L, T_J and T_H are the temperatures at the cold end, the joint and the warm end, respectively.

For the HTS part of the lead, the heat current is constant along the axis and is identical to the cooling load at the cold end, since no heat is generated.

$$Q_L = Q_1 = \frac{A_1}{L_1} \int_{T_L}^{T_J} k_1 \cdot dT \qquad (1)$$

where A and L are the length and the cross-sectional area of the lead, respectively and k is the thermal conductivity.

For an infinitesimal length of the metal lead shown in Figure 2, the energy balance equation can be written as $dQ = -dQ_{gen}$ and the heat generation rate, dQ_{gen}, is expressed by combining the one-dimensional equations for the Fourier heat conduction and the Ohmic heat generation.

$$dQ = -dQ_{gen} = -\frac{\rho_2 k_2 I^2}{Q} dT \qquad (2)$$

where ρ is the electrical resistivity and I is the current that the lead is carrying. After Equation (2) is multiplied by Q and integrated over the metal length, it can be rearranged for the heat current from the metal lead to the joint, Q_2.

$$Q_2 = \sqrt{Q_H^2 + 2I^2 \int_{T_J}^{T_H} \rho_2 k_2 \cdot dT} \qquad (3)$$

It is immediately observed that Q_2 has its minimum when the heat current at the warm end, Q_H, is zero. If Q_H has a positive value, Q_2 is larger than the minimum because of the excessive heat conduction through the metal lead. If Q_H has a negative value on the contrary, Q_2 is also

larger because of the excessive heat generation. The axial temperature gradient should be zero at the warm end when the heat conduction and the heat generation are optimally balanced. This condition for the minimum is identical to the case of the conventional vapor-cooled metal lead[12].

The minimum heat current to the joint is now found in a closed form by letting $Q_H = 0$.

$$(Q_2)_{min} = I \sqrt{2 \int_{T_J}^{T_H} \rho_2 k_2 \cdot dT} \tag{4}$$

Thus the minimum cooling load at joint is obtained from Equations (1) and (4).

$$(Q_J)_{min} = I \sqrt{2 \int_{T_J}^{T_H} \rho_2 k_2 \cdot dT} - \frac{A_1}{L_1} \int_{T_L}^{T_J} k_1 \cdot dT \tag{5}$$

At any arbitrary axial location of the metal lead, the optimal heat current is found as a function of temperature by integrating Equation (2) from the axial location to the warm end and letting $Q_H = 0$.

$$Q_{opt}(T) = I \sqrt{2 \int_{T}^{T_H} \rho_2(\tau) k_2(\tau) \cdot d\tau} \tag{6}$$

from which the optimal dimensions of the metal lead are found.

$$\left(\frac{L_2}{A_2}\right)_{opt} = \int_{T_J}^{T_H} \frac{k_2}{Q_{opt}(T)} \cdot dT = \frac{1}{\sqrt{2} \cdot I} \int_{T_J}^{T_H} \frac{k_2}{\sqrt{\int_{T}^{T_H} \rho_2(\tau) k_2(\tau) \cdot d\tau}} \cdot dT \tag{7}$$

It should be noted that the optimal conditions for the metal lead are related to the joint temperature of the two parts but are independent of the dimensions the HTS lead.

Equations (4) and (7) are very simple and useful expressions for the conduction-cooled metal lead, which have been reported recently by the present authors[11]. Similar optimal conditions were reported in previous publications with simple assumptions for the material properties, which could be derived as special cases of these general expressions. If the electrical resistivity and the thermal conductivity are assumed to be constant, Equation (4) is reduced to

$$(Q_2)_{min} = I \sqrt{2 \rho_2 k_2 (T_H - T_J)} \tag{8}$$

which was described by Seol and Hull[5]. For materials that obey the Wiedemann-Franz law, $\rho \cdot k = L_0 T$, Equation (4) is directly integrated to

$$(Q_2)_{min} = I \sqrt{L_0 \left(T_H^2 - T_J^2\right)} \tag{9}$$

as Yang and Pfotenhauer[6] mentioned.

REFRIGERATOR POWER

The total power input to a cryocooler for the two cooling loads can be generally expressed as,

$$W_{ref} = \left(\frac{T_H}{T_L} - 1\right) \cdot \frac{Q_L}{FOM_L} + \left(\frac{T_H}{T_J} - 1\right) \cdot \frac{Q_J}{FOM_J} \tag{10}$$

where FOM is the figure of merit, defined as the ratio of the actual coefficient of performance (COP) to Carnot's coefficient of performance. In Equation (10), it has been assumed that the warm end temperature, T_H, is identical to room temperature at which the cryocooler rejects heat. The FOM of a cryocooler depends on the cold head temperature, the type of refrigeration cycle, the refrigeration capacity, the performance of its components, and so on.
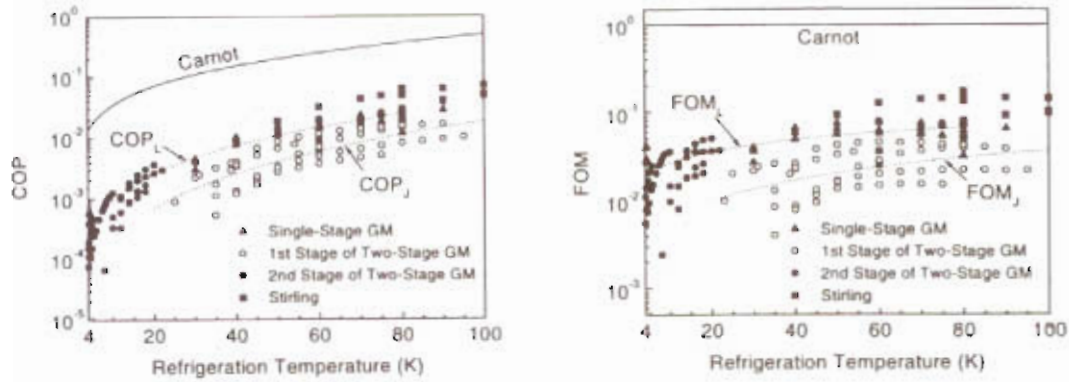
Figure 3. *COP* (coefficient of performance) and *FOM* (figure of merit) of some commercial cryocoolers and the refrigeration models in current study.

If the *FOM's* are known for a specific cryocooler, the refrigerator power can be calculated from Equation (10). On the other hand, for the purpose of a more quantitative demonstration of the optimization, it has been suggested in this study to construct a simple yet reasonable model to the *FOM's* depicting the performance of actual cryocoolers. An extensive survey has been executed on the performance of commercial cryocoolers that can be applied to the cooling of the binary current leads, and the *COP* and the *FOM* are plotted as functions of the refrigeration temperature in Figure 3. It is noticed that the *COP* and the *FOM* of the second stage of two-stage GM cryocoolers or the single-stage coolers have greater values than those of the first stage of two-stage GM cryocoolers. In most practical cases, the cold end of the binary lead is cooled by the second stage of a two-stage cooler and the joint of the two parts are cooled by the first stage, so the two *FOM's* are expressed by simple functions,

$$FOM_L = \frac{0.1202 \cdot T_L + 1.316}{T_L + 84.81} \quad \text{for } 4 \text{ K} \leq T_L \leq 40 \text{ K}$$

$$FOM_J = \frac{0.4198 \cdot T_J + 2.740}{T_J + 1115} \quad \text{for } 20 \text{ K} \leq T_J \leq 100 \text{ K}$$

(11)

as indicated by two dashed curves in Figure 3. In this model, the FOM_L has a value between 0.02 and 0.05, and FOM_J has a value between 0.01 and 0.035 in the valid temperature ranges.

OPTIMIZATION

The integration of the binary current lead and the two-stage cryocooler can be optimized such that the total refrigerator power per unit current[1-3,6,9] has a minimum. The mathematical expression for the refrigerator power per unit current is derived by substituting Equations (1) and (4) into Equation (10) and dividing it by *I*.

$$\frac{W_{ref}}{I} = \frac{1}{FOM_L}\left(\frac{T_H}{T_L}-1\right)\frac{1}{J_1 L_1}\int_{T_L}^{T_J} k_1 \cdot dT + \frac{1}{FOM_J}\left(\frac{T_H}{T_J}-1\right)\left(\sqrt{2\int_{T_J}^{T_H}\rho_2 k_2 \cdot dT} - \frac{1}{J_1 L_1}\int_{T_L}^{T_J}k_1 \cdot dT\right)$$

(12)

where J_1 is the current density at the HTS. It is worthwhile to notice in Equation (12) that the power per unit current is a function of J_1, L_1 and T_J only, when the *FOM's*, the material properties and the end temperatures are given. The readers may be reminded that J_2 and L_2 are not included in Equation (12) because they have been already optimized as in Equation (7).

The refrigerator power per unit current has been calculated with Equation (12) for various values of the current density and the joint temperature in a copper+Bi2223 binary lead. Figure 4 shows contours of the refrigerator power per unit current on a current density (J_1) vs. joint
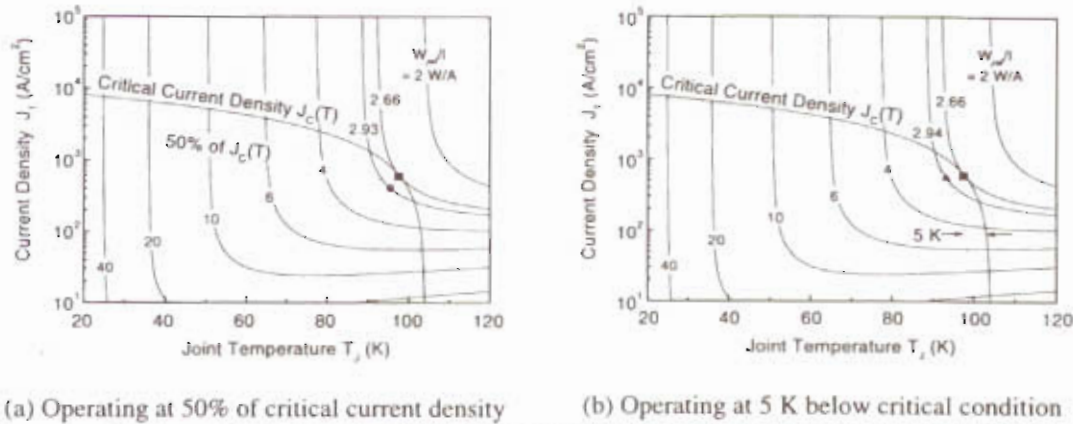
(a) Operating at 50% of critical current density   (b) Operating at 5 K below critical condition

Figure 4.   Contours of refrigerator power per unit current and critical current density of HTS on current density (J_1) and joint temperature (T_1) diagram of Cu+Bi2223 when L_1 =20 cm .

temperature (T_J) diagram for $L_1 = 20$ cm and *FOM's* given by Equation (11). The thermal conductivity of Bi2223 is taken from Herrmann et al.[3] and the properties of copper are taken from Maehata et al.[13] for RRR=60. The cold and warm end temperatures of the lead are fixed at 4 K and 300 K, respectively, throughout this paper.

Generally speaking, the refrigerator power per unit current decreases as the current density of the HTS or the joint temperature increases. However, when the joint temperature is low or the current density is high, the power per unit current is almost independent of the current density since the cooling load for the HTS or the first term in Equation (12) is relatively small. On the contrary, when the joint temperature is high or the current density is low, the first term is dominant and the current density is relatively more significant in the total power per unit current than the joint temperature.

Since Equation (12) has been derived with the assumption that the HTS does not generate heat, the superconductivity should be confirmed by incorporating the critical properties of the HTS, which will establish the final process of the optimization. The current density of the HTS should not exceed the critical current density, which can be represented reasonably well by a linear function of temperature for Bi2223 [1,4].

$$J_1 \le J_C = J_{C0}\left(1 - \frac{T}{T_C}\right) \tag{13}$$

where J_{C0} is the critical current density at 0 K and varies over a wide range, depending upon the size, the shape and the fabrication method as well as the applied magnetic field. For the purpose of more quantitative discussions in this paper, it is assumed that $J_{C0} = 10,000$ A/cm^2 and $T_C = 104$ K for Bi2223 at zero magnetic field[4] and Equation (13) is plotted on the J_1-T_J diagram of Figure 4. Clearly, there exist unique optimal values for the current density and the joint temperature to minimize the refrigerator power per unit current while the HTS is superconducting, as indicated by the square dot. At higher joint temperatures and smaller current densities than the optima, more refrigerator power per unit current is required because of a greater refrigerator power to cool the HTS. At lower joint temperatures and larger current densities, more power per unit current is also required because of a greater power to cool the metal part of the lead. The concave shape of the curves at low J_1 region indicates that for a constant current density, the optimum joint temperature to minimize the power per unit current should be significantly lower than the critical temperature, as discussed by Yang and Pfotenhauer[6].

The theoretical minimum of the power per unit current is about 2.66 W/A with the present cryocooler model, which is about 29 times greater than the minimum for a two-stage Carnot refrigerator[11] and about 47 times the absolute minimum as a thermodynamic limit[11] that can be

obtained with distributed Carnot refrigerator. The corresponding optimal values of J_l and T_J are 610 A/cm^2 and 97.7 K, respectively.

Because of the nature of the superconductivity, the theoretical optimum to minimize the refrigerator power per unit current is always determined at a marginally superconducting state as shown in Figure 4. In practice, the HTS current leads should be designed such that the current density and the joint temperature are lower than their theoretical optima in order to be stable from a certain level of thermal disturbance. Two different design schemes are suggested for the determination of the actual operating conditions with some stability margins. The first one is to find the operating condition among the points where the operating current density of the HTS has a certain fraction of its critical value at any arbitrary temperature. The dotted curve on Figure 4(a) shows the points where J_l is 50 % of $J_C(T_J)$ and the refrigerator power per unit current on the curve has a unique minimum, 2.93 W/A as indicated by a circle dot. This design is based on a safety factor of 2 in the operating current density. The corresponding optima for J_J and T_J are 415 A/cm^2 and 95.4 K, respectively. The second design scheme is to find the operating condition among the points where the joint temperature is lower by a certain amount than its critical temperature at any arbitrary current density. The temperature difference is related closely with the magnitude of the disturbance energy to initiate a quench in the HTS lead or to propagate the normal zone. The dotted curve on Figure 4(b) shows the points where T_J is lower by 5 K than the critical curve. The refrigerator power per unit current on the curve has a unique minimum, 2.94 W/A as indicated by a triangle dot, when J_l and T_J are 551 A/cm^2 and 93.3 K, respectively. In this specific example, the optimally designed refrigerator power has about 10% more than the theoretical minimum in the both cases.

The above procedures have been repeated for various values of the HTS length, L_l, and the results have been plotted in Figures 5 through 7. In these figures, the theoretical optima are marked by the squares and the two suggested designs are marked by the circles and the triangles, respectively, as in Figure 4. It is observed in Figure 5 that as L_l increases, the refrigerator power per unit current decreases for every design. However, the power per unit current does not vary significantly if L_l is greater than about 10 cm, which means that the length of the HTS does not need to be very long as far as it is optimally cooled. As L_l decreases to zero, the power per unit current approaches the values required by single-stage cooling of an optimized metallic lead. If L_l is infinitely large, all contours of W_{ref}/I in Figure 4 are vertical and the theoretical power per unit current will be reduced to 1.99 W/A, which can be directly calculated by the asymptotic behavior of Equation (12).

$$\left(\frac{W_{ref}}{I}\right)_{min} = \frac{1}{FOM_J(T_C)}\left(\frac{T_H}{T_C} - 1\right)\sqrt{2\int_{T_C}^{T_H}\rho_2 k_2 \cdot dT} \tag{14}$$

In this simple case, the cooling load at the cold end is negligible and the joint temperature is the critical temperature of the HTS.
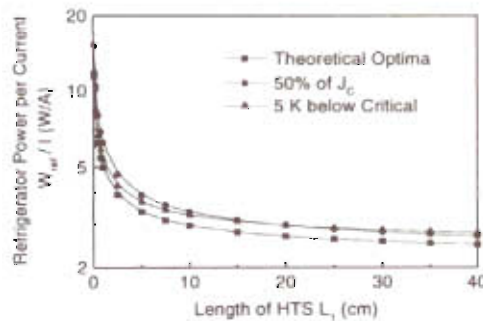

Figure 5. Optimized refrigerator power per unit current as a function of HTS Length for Cu+Bi2223.
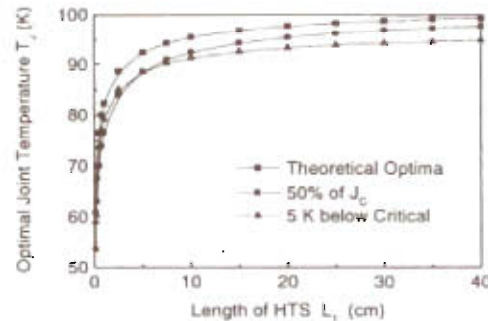
Figure 6. Optimal joint temperature as a function of HTS Length for Cu+Bi2223.
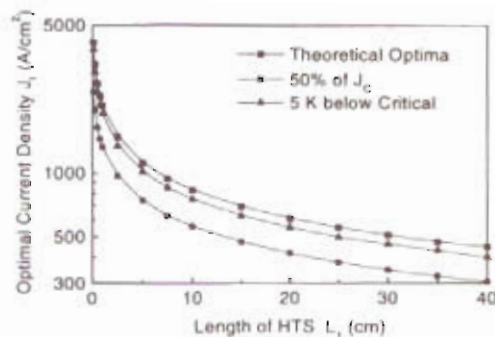
Figure 7. Optimal current density of HTS as a function of HTS Length for Cu+Bi2223.
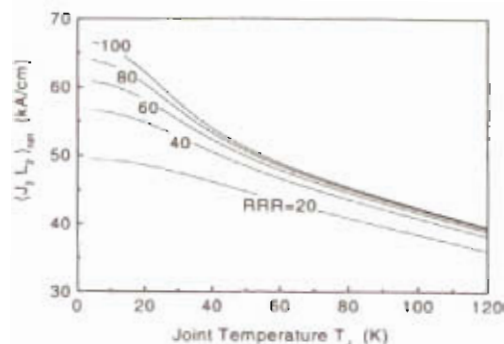

Figure 8. Optimal current density and length of metal (copper) as a function of joint temperature for various RRR's.

In Figure 6, the optimal T_J increases and approaches to the critical temperature of the HTS, as L_I increases. It is noted that if L_I is 10 cm or 30 cm as is typical of commercial HTS lead, the practical design values of T_J are found in a range between 90 K and 95 K, while the theoretical optima are slightly higher. It is observed in Figure 7 that as L_I increases, the optimal values of J_I decreases monotonically and the optima including the stability margins have smaller values than the theoretical optima.

Once the joint temperature is optimally determined, the design for the metal lead should be finally designed by Equation (7). Figure 8 shows the optimal product of the current density and the length of the metal lead as a function of the joint temperature for various RRR values of copper. The properties of copper have been taken from Maeheta et al.[13], again. It should be kept in mind that if the material and the end temperatures of the metal lead are given, there are an infinite number of combinations for J_I and L_I to minimize the cooling load, but the minimum load is the same for every optimized combination, as given by Equation (4).

The theoretical optimization and the suggested designs for Bi2223+Cu leads are summarized and compared with the corresponding thermodynamic limits[11] in Table 1. The optimized results for $L_I=0$ are the cases of the metallic lead, in which the optimal single-stage cooling at the cold end can be directly derived by letting $T_J = T_L$ in Equations (4) and (7). When $L_I = \infty$, the cooling load at the cold end is negligible and the optimal joint temperature is the critical temperature of the HTS as described by Equation (14). It is noted that the optimal values of $J_2 L_2$ for the metal lead are not strongly dependent on the cooling method or the design scheme, because the optimal joint temperature does not vary significantly in this specific example.

A NEW DIMENSIONLESS NUMBER

In the previous section, the optimization for the cryocooler-cooled binary lead has been demonstrated with the variables having real dimension. It is worth performing a simple dimensional analysis, because the number of independent variables may be reduced in the optimization process.

The minimum of the refrigerator power per unit current, Equation (12), with a constraint, Equation (13), can be expressed in general as a function of the two end temperatures, the critical properties and the length of the HTS lead, the thermal conductivity and the electrical resistivity for two materials as functions of temperature, and the performance of the cryocoolers for any optimization scheme.

$$\left(W_{ref}/I\right)_{min} = f\left[T_L, T_H, T_C, J_{C0}, L_1, k_1(T), \rho_2(T), k_2(T), FOM_J, FOM_L\right] \qquad (15)$$

Equation (15) can be written in a dimensionless form as

Table 1. Summary of Given Conditions and Optimized Results

(1) Given Conditions

End temperatures	Warm end	T_H	300 K	
	Cold end	T_L	4 K	
Lead materials	HTS	1	Bi2223 (T_C=104 K, J_{C0}=10,000 A/cm^2) Thermal conductivity from Herrmann et al.[3]	
	Metal	2	Copper (RRR=60) Thermal conductivity & electrical resistivity from Maehata et al.[13]	

(2) Optimized Results

Length of HTS	Optimized values	Distributed Carnot*	Two-stage Carnot*	Two-stage cryocooler FOM's given by Equation (11)		
				Theoretical	50 % of J_C	5 K below
$L_1 = 0$ (All metal)	W_{ref}/I (W/A)	0.222	3.10**	153**	153**	153**
	Q_L/I (W/A)	-	0.0419	0.0419	0.0419	0.0419
L_1 =20 cm	W_{ref}/I (W/A)	0.0569	0.0907	2.66	2.93	2.94
	T_J (K)	99	98	97.7	95.4	93.3
	J_1 (A/cm^2)	481	577	610	415	551
	Q_J/I (W/A)	-	0.0405	0.0404	0.0404	0.0406
	Q_L/I (mW/A)	-	0.0975	0.0917	0.130	0.0953
	$J_2 L_2$ (kA/cm)	43.3	41.9	42.0	42.3	42.6
$L_1 = \infty$	W_{ref}/I (W/A)	0.0516	0.0759	1.99	1.99	2.25
	Q_J/I (W/A)	-	0.0403	0.0403	0.0403	0.0404

* Provided by Chang & Van Sciver[11]

** Single-stage cooling at 4 K

$$\frac{\left(W_{ref}/I\right)_{min}}{\sqrt{T_C \rho_2(T_C)k_2(T_C)}} = f\left[\frac{T_L}{T_C}, \frac{T_H}{T_C}, \frac{k_1(T)}{k_1(T_C)}, \frac{\rho_2(T)}{\rho_2(T_C)}, \frac{k_2(T)}{k_2(T_C)}, FOM_J, FOM_L, CV\right] \quad (16)$$

where a significant dimensionless variable is defined as

$$CV \equiv \frac{J_{C0}L_1}{k_1(T_C)}\sqrt{\frac{\rho_2(T_C)k_2(T_C)}{T_C}} \quad (17)$$

and will be called the CV number after the present authors.

The *CV* number is composed of the properties of the HTS and the metal and the length of the HTS, but has nothing to do with the cooling method. The number is physically interpreted as the relative magnitude of the cooling load of the optimized metal lead with respect to the HTS lead. It is obvious that the cooling load of the HTS lead can be reduced by increasing the current density or the length as they are in the numerator of Equation (17), or by decreasing the thermal conductivity as it is in the denominator. It is also noted that the square root term in the *CV* number is a constant for metals that obey the Wiedemann-Franz law. The value of the *CV* number is approximately 1,550 for the Bi2223+Cu lead when $J_{C0} = 10,000$ A/cm^2 and $L_1 = 20$ cm.

An evident usefulness of the *CV* number can be illustrated with Figures 5 through 7. The graphs for the optimal conditions have been generated as functions of L_1 when $J_{C0} = 10,000$ A/cm^2. As mentioned earlier, J_{C0} depends upon the size, the shape, the fabrication method and the applied magnetic field. Even if J_{C0} of the available HTS lead may not be 10,000 A/cm^2, Figures 5 through 7 could be used to determine the optimal values for $W_{ref}/I\sqrt{T_C \rho_2(T_C)k_2(T_C)}$, T_J/T_C, and J_1/J_{C0}, with an equivalent L_1 that yields the same *CV* number.

SUMMARY

A complete design method is developed for the optimal cooling of the binary current lead

with a two-stage cryocooler. The optimization aims at the minimum of the required power input per unit current, provided that the critical properties and the length of the HTS lead, the refrigeration performance of the cryocooler and the two end temperatures of the binary lead are given. The analytical or graphical design procedure can be summarized as the following steps.

(1) The required refrigerator power input is calculated by Equation (12) with an assumption that the dimensions of the metal lead are optimal.

(2) The optimal values for the joint temperature and the current density of the HTS should be determined "simultaneously" by minimizing Equation (12) with a constraint, Equation (13) as in Figure 4. A proper stability margin may be included in this step.

(3) With the optimal joint temperature, the product of the length and the current density of the metal lead is optimized by Equation (7) as in Figure 8. Any combination of the length and the current density will result in the same cooling load for the metal.

ACKNOWLEDGMENT

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