OPTIMAL DESIGN OF SIZES IN BINARY CURRENT LEADS COOLED BY CRYOGENIC REFRIGERATOR

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ABSTRACT

Analysis is performed to determine the optimal lengths or cross-sectional areas of refrigerator-cooled current leads that can be applied to the conduction-cooled superconducting systems. The binary current lead is composed of the series combination of a normal metal at the upper (warm) part and a high Tc superconductor (HTS) at the lower (cold) part. The heat conduction toward the cold end of HTS part constitutes a major refrigeration load. In addition, the joint between the two parts should be cooled by a refrigerator in order to reduce the load at the low end and maintain the HTS part in a superconducting state. The sum of the work inputs required for the two refrigeration loads needs to be minimized for an optimal operation. In this design, three simple models that depict the refrigeration performance as functions of cooling temperature are developed based on some of the existing refrigerators. By solving one-dimensional conduction equation that takes into account the temperature-dependent properties of the materials, the refrigeration works are numerically calculated for various values of the joint temperatures and the sizes of two parts. The results show that for a given size of HTS, there exist the optimal values for the joint temperature and the size of the normal metal. It is also found that the refrigeration work decreases as the length of HTS increases and that the optimal size of the normal metal is quite independent of the size of HTS. For a given length of HTS, there is an optimal cross-sectional area and it increases as the length increases. The dependence of the optimal sizes on the refrigerator models employed are presented for 1 kA lead.

INTRODUCTION

Current leads are key components in superconducting systems that supply electrical current to the cold superconducting magnet from the power source at ambient temperatures. The material for an ideal current lead should have a good electrical conductivity and a poor thermal conductivity, since both the Joule heat generation in the lead and the heat

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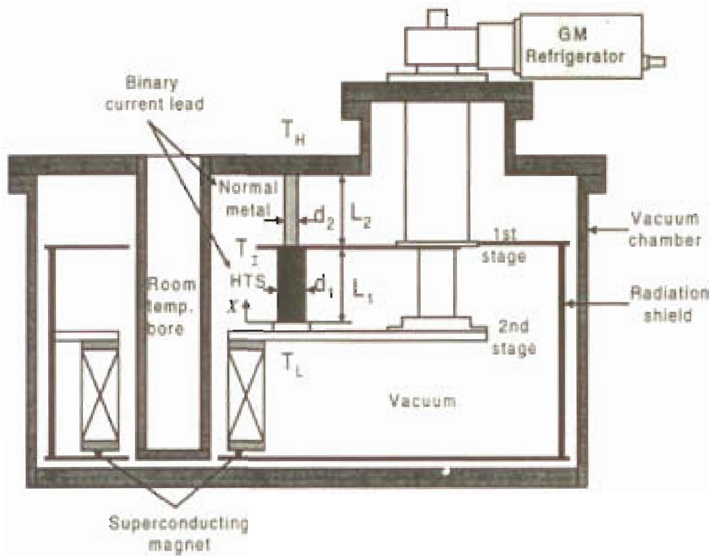


Figure 1. Schematic representation of refrigerator-cooled superconducting systems with binary (higher-temperature metal and lower-temperature HTS) current leads.

conduction through the lead to the cold magnet should be small. The conventional metallic leads[1-2], which basically obey the Wiedemann-Franz law, is constrained by an inherent limit in reducing the heat leak to the cold magnet. In practice, the largest cooling load of the conventional superconducting systems is due to the metallic current leads.

After the discovery of the high Tc superconductors (HTS), the earliest useful application has been as a low temperature component of the current lead. The oxidized ceramic superconductors have very low thermal conductivity and zero electrical resistance as long as they remain in a superconducting state. The binary or hybrid current lead[3-10] is the series combination of normal metal as a higher temperature part and HTS as a lower temperature part. Over the past several years, there have been many efforts directed at the practical utilization of the binary leads.

In addition, recent progress in the development of 4 K Gifford-McMahon refrigerators[11] has raised the possibility of the liquid-free or the refrigerator-cooled superconducting magnets[12-13]. Since there is no liquid helium, one of the most significant and inevitable problems in the refrigerator-cooled superconducting system is the cooling of the current lead without helium vapor in vacuum. A typical configuration of the liquid-free superconducting systems is schematically shown in Figure 1. The second stage of a 2-stage GM refrigerator cools the magnet and the cold end of the binary current leads at about 4 K, while the first stage cools the joint between the two parts of the binary lead and the radiation shield. It is a well-established fact that the intermediate cooling is necessary to reduce the amount of heat leak to the lower temperature as well as to maintain the HTS in a superconducting state. The authors think that the feasibility of the refrigerator-cooled current leads has been demonstrated by recent studies[12-13], and the next crucial step towards the practical application could be to develop energy-efficient current leads.

In a refrigerator-cooled current lead, there are two separate cooling loads at respective temperatures, and the optimal design with respect to the energy efficiency can be obtained by minimizing the sum of the work inputs for the two refrigeration loads. This study aims to determine the optimal conditions for 1 kA level of binary current leads, including the intermediate cooling temperature and the sizes (the cross-sectional area and the length) of both parts of the current lead. The analysis incorporates the temperature-dependent material properties and is based upon a few simple yet reasonable models of the refrigeration performance.

ANALYSIS MODEL

Cooling Load

The temperature of the current lead in vacuum as shown in Figure 1 can be determined by solving one-dimensional conduction equation

$$\frac{d}{dx}\left(k(T)\frac{dT}{dx}\right)+\frac{\rho(T)I^2}{A}=0 \tag{1}$$

where $T(x)$ is the temperature along the lead and A is the cross-sectional area. The thermal conductivity, $k(T)$, and the electrical resistivity, $\rho(T)$, are functions of temperature. The coordinate x is defined as the distance from the cold end of the lead. Eq.(1) can be applied both to the metal and the HTS. For the boundary conditions, the warm end of the normal metal part operates at the room temperature, T_H, and the cold end of the HTS part operates at the low temperature, T_L, which is identical to the magnet temperature. At the joint, the contact resistance is neglected, so that the two parts are assumed to have the same temperature, T_I. The cooling load at the joint can be expressed as

$$Q_I=k_2(T_I)A_2\frac{dT_2(L_1)}{dx}-\left(k_1(T_I)A_1\frac{dT_1(L_1)}{dx}\right) \tag{2}$$

and the cooling load at the cold end can be expressed as

$$Q_L=k_1(T_L)A_1\frac{dT_1(0)}{dx} \tag{3}$$

In Eqs.(2) and (3), the subscripts 1 and 2 denote the HTS and the normal metal, respectively.

Properties of Materials

The thermal conductivity and the electrical resistivity of normal metals are strongly dependent on temperature. The Wiedemann-Franz law describes the general correlation of the two properties with temperature. In this paper, however, the functional relations for the two properties given by White[2] and Jones et al.[4] are selected for a more precise calculation of the temperature distribution.

Concerning the high Tc superconductors, two properties are important for this study. The first one concerns the critical property or the normal-superconducting transition property. Typically the transition property is given by the current density (J_c)-temperature (T)-magnetic field (B) surface[14]. The detailed phenomena are quite complicated and depend on the composition of the materials and the fabrication processes. In this study, a simple model with a linear J_c-T relation for zero magnetic field is selected. For HTS, the electrical resistivity in Eq.(1) is assumed to be zero, as long as the HTS material is superconducting. If this condition is satisfied, Eq.(1) is directly integrated so that the cooling load at the cold end may be written as

$$Q_L = \frac{A_1}{L_1}\int_{T_L}^{T_I} k_1(T)dT \tag{4}$$

The second property concerns the thermal conductivity of HTS as a function of temperature. A functional relation is taken from Herrmann et al.[5] for BSCCO-2212.

Refrigerator

The most difficult and crucial part of the analysis model concerns the performance of the refrigerators. As mentioned above, in the binary current leads, there are two cooling loads at two different temperatures. Generally, the work required for refrigeration depends on the refrigeration temperature and the refrigerator type as well as on the cooling load. The sum of the works for the two cooling loads rather than the sum of the two cooling loads should be minimized for an optimal operation. The refrigeration work can be expressed as

$$\begin{aligned} W &= W_I + W_L \\ &= \frac{Q_I}{FOM_I}\left(\frac{T_H}{T_I}-1\right) + \frac{Q_L}{FOM_L}\left(\frac{T_H}{T_L}-1\right) \end{aligned} \tag{5}$$

where *FOM* denotes the figure of merit, defined as the ratio of the coefficient of performance (*COP* = refrigeration/work) of the actual refrigerator to that of reversible refrigerator which operates between the same refrigeration temperature and the same heat rejection temperature. In Eq.(5), the temperature at which the heat is rejected from the refrigerator is assumed to be the room temperature, T_H.

To construct simple yet reasonable models for the FOM_I of the refrigerator, the performance of typical commercial refrigerators are surveyed and plotted in Figure 2. It is observed that for Stirling refrigerators, the COP and the FOM are strongly dependent on the size or the cooling capacity as well as on the refrigeration temperature. The required cooling capacity is closely related with the current level of the lead. For GM (Gifford-McMahon) refrigerators, however, the COP and the FOM are determined primarily by the refrigeration temperature and have values close to those for the small sized Stirling refrigerators. It is also noticed that the FOM of GM refrigerators decreases as the refrigeration temperature rises

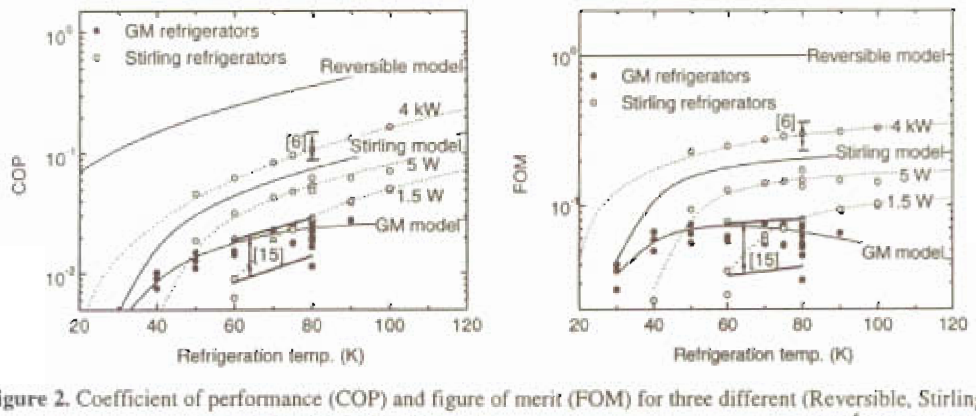


Figure 2. Coefficient of performance (COP) and figure of merit (FOM) for three different (Reversible, Stirling, Gifford-McMahon) refrigerator models as functions of refrigeration temperature. Hermann et al.[6] have assumed greater FOM's than practical refrigerator and Yang and Pfotenhauer[15] have assumed FOM's close to those of GM model.

above 80 K. No commercial pulse tube refrigerator could be found for the capacity and temperature range of interest.

In this paper, three different refrigerator models - reversible, Stirling, and GM - are considered as shown by solid curves in Figure 2. The reversible refrigerator model represents the thermodynamically ideal refrigeration or the case of the minimal work required for a given refrigeration load. The Stirling refrigerator model is selected for approximately 40 W range at 80 K, which is suitable for 1 kA binary leads of the optimal design case to be described below. The GM refrigerator model is closer to many commercial refrigerators. It is noted that the FOM of Stirling model is higher than that of GM model at relatively higher temperatures and nearly the same at relatively lower temperatures. Hermann et al.[6] have assumed higher FOM's than the typical commercial refrigerator and Yang and Pfotenhauer[15] have assumed FOM's close to those of GM model.

Since the cold end temperature of the binary current lead is fixed and is not a design parameter, the FOM_L of the refrigerator for cooling the cold end is assumed to be a constant. Since a typical value of FOM_L of the refrigerator at about 4 K is 1/15[16-17], it is used in the calculation. When a two-stage refrigerator cools the current lead as shown in Figure 1, the refrigeration performance at the second stage is related with that at the first stage in a rather complicated manner. In the present analysis model, the two performances are assumed to be independent for simplicity.

RESULTS AND DISCUSSION

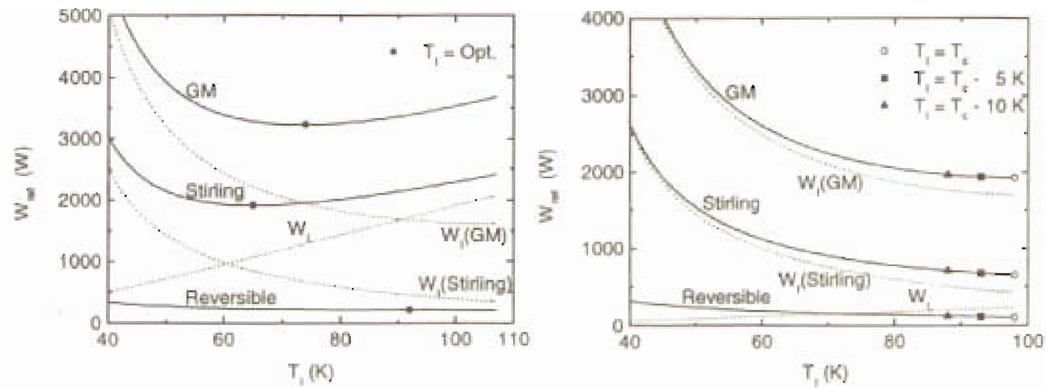
Intermediate Cooling Temperature

The intermediate cooling temperature, T_I, at which the joint of the normal metal and the HTS is maintained, is important in designing the binary current lead. For all cases considered in this paper, the materials for the normal metal and the HTS in this analysis are copper and BSCCO-2212, respectively. The room temperature and the cold end temperature are 300 K and 4.2 K, respectively. The current level of the lead is 1 kA, unless specified otherwise.

Figure 3 shows the total refrigeration work as functions of the intermediate cooling temperature for three refrigeration models, all for specific sizes of the lead. The solid curves denote the sum of work for the two loads, which are shown as the dotted curves. The curves for the total work can be classified into two difference shapes; typical representations are given in Figures 3(a) and (b).

For Figure 3(a), the HTS is relatively thick and short so that the refrigeration work for the cold load is comparable with that for the intermediate load. It is clearly observed that for each model, there exists an optimal T_I that minimizes the refrigeration work. At T_I lower than the optimum, the total work is larger due to the work for the cold end load, and at higher T_I, the total work is also larger due to the work for the intermediate cooling load. In this case, it is noted that the optimal value of the intermediate cooling temperature is about 92 K for the reversible model, but is as low as about 73 K or 65 K for Stirling or GM model. The reason why the Stirling or GM model has a lower optimum than the reversible model is that the FOM_L in the Stirling or GM model is much lower, and the work (W_L) for the cold end cooling is more significant than in the reversible model. Apparently, the highest possible value of the intermediate cooling temperature is limited by the critical current density of HTS.

For the case of Figure 3(b), the HTS is relatively thin and long so that the refrigeration work for the cold load is negligible. It is observed that the total refrigeration work decreases as the intermediate temperature increases to the critical temperature. From the viewpoint of



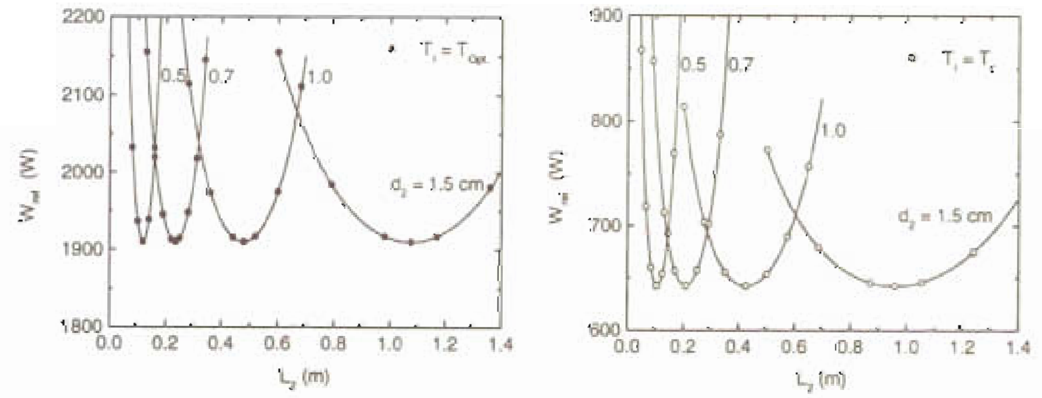
(a) d_1= 4 cm, L_1= 10 cm (b) d_1= 2 cm, L_1= 20 cm

Figure 3. Refrigeration work as a function of intermediate cooling temperature for three refrigerator models. (1: BSCCO-2212, 2: Cu, I= 1 kA, d_2= 0.5 cm, L_2= 12 cm)

energy efficiency, the optimal T_1 is the highest possible value that still keeps the HTS superconducting. In practice, when we consider the thermal stability of HTS, T_1 should be set at a temperature slightly lower than the critical temperature. The points where T_1 is lower than T_c by 5 K or 10 K are indicated on the curves. The four different markers in the legends in Figure 3 will be used consistently in the subsequent graphs. It should be noted that since the analysis is one-dimensional, the diameter of the lead simply represents the size of the cross-section and the lead should not necessarily be a circular rod.

Sizes of Normal Metal

Figure 4 shows the total refrigeration work as a function of the metal diameter for various values of the metal length. In the case of Figure 4(a), the intermediate cooling temperature has been optimized as in Figure 3(a), and in the case of Figure 4(b), the intermediate cooling temperature is the critical temperature of Figure 3(b). In both cases, it is noted that for a fixed diameter, the length has an optimum value with respect to the minimum refrigeration work required. When the length is shorter than the optimum, the cooling load due to the Joule heating by large electrical resistance entails more work. When the length is longer, the load due to large thermal conduction also causes more work.



(a) d_1= 4 cm, L_1= 10 cm, optimal T_1 (b) d_1= 2 cm, L_1= 20 cm, T_1 = T_c

Figure 4. Refrigeration work as a function of diameter of normal metal for various lengths of normal metal. (1: BSCCO-2212, 2: Cu, I= 1 kA, Stirling model)

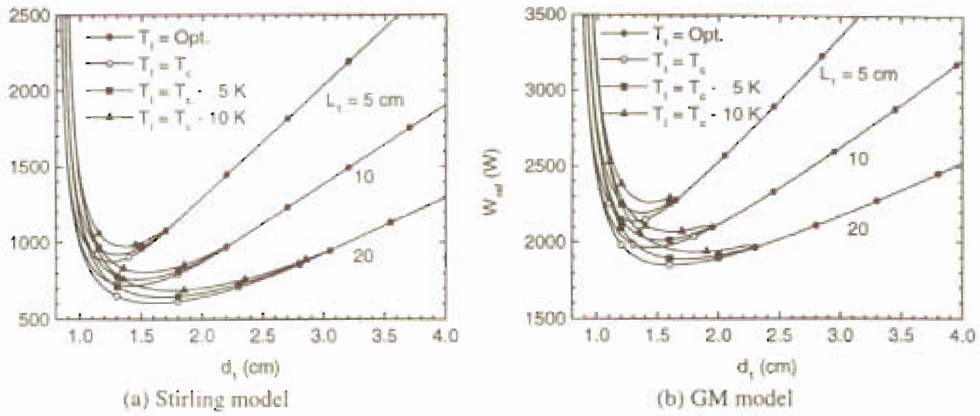


Figure 5. Refrigeration work as a function of diameter of HTS for various values of HTS length with Stirling and GM models. The markers indicates how the joint (intermediate cooling) temperatures are selected.
(1: BSCCO-2212, 2: Cu, d_2 & L_2=optimized, I= 1 kA)

It is interesting to note that the optimal length increases but the refrigeration work remains nearly constant as the diameter increases. This behavior is quite similar to the case of the simple current lead without HTS. In practice, the length of the normal metal will be constrained by the geometric factors such as the length of the cold head of the refrigerator. With the constraints included, the optimal diameter is determined such that the total refrigeration work is minimized. While there is a close relationship between the length and the diameter of the normal metal for minimum work, the minimum work itself is dependent not on the sizes, but only on the current level. It should also be mentioned that in practical design, a thick and long design of the metal part may be desirable, because the additional work may not entail large cost even if the design deviates from the optimal.

Sizes of HTS

The sizes of HTS part are important design factors in the practical design of the binary current lead. In particular, the sizes are quite closely related with the fabrication method of HTS. Concerning the normal metal, the sizes are important from the viewpoint of balancing the heat generation by electrical resistance and the heat conduction. However, since the HTS has no electrical resistance in superconducting state, no heat is generated and the heat conduction becomes the dominant factor in minimizing the refrigeration work. In general, a large length or small diameter of the HTS part will be preferred so far as the refrigeration is concerned. On the other hand, the cross sectional area of the HTS affects the current level which the lead can carry in a superconducting state or the temperature level below which the HTS should be cooled to.

Figure 5 shows the refrigeration work as a function of HTS diameter given various values of HTS length for Stirling and GM refrigerator models. In the calculations, the sizes of the normal metal and the intermediate cooling temperature have been optimized as described in the previous sections. It is noted that if the diameter is large enough, there exists an optimal T_I as in Figure 3(a). As the diameter decreases, the optimal T_I reaches the critical temperature and then the T_I can be chosen as in Figure 3(b).

Regardless of the choice of T_I, it is found that the refrigeration work decreases as the length of HTS increases. For a given length, however, there is an optimal diameter of the HTS that requires a minimum refrigeration work. As the diameter increases above the optimum, the heat conduction through the HTS to the cold end increases, and the corresponding refrigeration work increases as well. As the diameter decreases, the current density increases and the optimal intermediate temperature reaches the critical temperature of HTS for the given current density. It is also noted that the refrigeration work is independent of the length when the diameter is much shorter than the optimum, and the optimal length-diameter combination does not significantly depend on the refrigeration models employed as well.

CONCLUSIONS

The refrigeration work required to cool 1 kA binary current leads is calculated through analysis to determine the effect of sizes. The optimal design that minimizes the work is also discussed. The calculation of the work is based upon three different simple models that depict the refrigeration performance as functions of cooling temperature. The following main conclusions can be drawn from the analysis.

(1) There exists an optimal intermediate cooling temperature between 60 K and 100 K in which the exact value depends on a given size combination of the metal and the HTS parts if the cold end load is relatively significant. When the cold end load is small, the work decreases as the intermediate cooling temperature increases up to the critical temperature of HTS.

(2) There exists an optimal the diameter-and-length combination of the metal part, if given an optimal intermediate cooling temperature and a particular size of HTS. As the length increases, the corresponding optimal diameter increases, but the refrigeration work does not vary much.

(3) When the intermediate cooling temperature and the size of the metal are optimally designed, the refrigeration work decreases as the length of HTS increases. For a given length of HTS, there exists a corresponding optimal diameter, and it increases with the length. It is demonstrated that in the case of 1 kA binary lead, the optimal diameters of HTS fall between 1.2 and 1.8 cm when the length are from 5 to 20 cm.

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