A Stirling Cycle Analysis with Gas-Wall Heat Transfer in Compressor and Expander

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ABSTRACT

A cycle analysis that includes the gas-wall heat transfer in compressors and expanders of Stirling cryocoolers is presented. The gas-wall heat transfer in the working spaces is quite different from the conventional convective heat transfer, since the pressure of the working gas inside the spaces oscillates and the heat transfer may not be in phase with the temperature difference between the gas and the wall. Several experimental and theoretical expressions have been published to date to estimate the amount of the gas-wall heat transfer and the so-called hysteresis loss for a gas spring. While most of the expressions for the gas spring are not directly applicable to the working spaces of Stirling coolers, some might be useful in predicting its effect on refrigeration.

The conventional adiabatic analysis for Stirling cycle is generalized by adding the heat transfer terms to the energy balance equations for the two working spaces. Of the several existing heat transfer relations for the gas spring, three different relations are used and compared. The results are verified by observing that the cycle asymptotically approaches the Schmidt isothermal limit for very small Peclet numbers and the Finkelstein adiabatic limit for very large Peclet numbers. It is found that the effect of the gas-wall heat transfer on the refrigeration would not be significant except for low-speed miniature coolers. The analyses are repeated for various frequencies, the bore/stroke ratios, the dead volumes and the phase angles between the two pistons, and their effects on the cooler performance are discussed.

NOMENCLATURE

A	Heat transfer area
COP	Coefficient of performance
D_h	Hydraulic diameter
H	Characteristic length of piston
f	Frequency
K	Coefficient defined by Eq.(6) or (7)
k	Thermal conductivity

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L	Stroke of piston
m	Mass
Nu_c	Complex Nusselt number
Nu_I	Imaginary part of Nu_c
Nu_R	Real part of Nu_c
P	Pressure
Pe	Peclet number
Pr	Prandtl number
Q	Heat transfer for a cycle
$\dot{Q}$	Heat transfer rate
R	Gas constant
Re	Reynolds number
T	Temperature
t	Time
V	Volume
W	Work for a cycle
z	Coefficient defined by Eq.(3)

Greek letters

α	Thermal diffusivity
ϕ	Phase angle advance
γ	Specific heat ratio
λ	Coefficient defined by Eq.(8)
ν	Kinematic viscosity
ζ	Coefficient defined by Eq.(8)

Subscripts

C	Compressor
E	Expander
H	Warm heat exchanger
L	Cold heat exchanger
R	Regenerator
$wall$	Wall temperature

INTRODUCTION

One of the most difficult problems in Stirling cycle analysis is to estimate the gas-wall heat transfer in working spaces. Concerning the heat transfer, there are two limiting analysis models - the isothermal (Schmidt) model and the adiabatic (Finkelstein) model.

In the isothermal analysis, the gas-wall heat transfer in the compressor and in the expander is assumed to be quite rapid so that the gas temperatures in the spaces are constant. The isothermal heat transfer represents a reversible process and always results in the maximum efficiency of the cycle. This model is valid for an extremely low speed operation or when extremely small wall-to-wall distances in the working spaces are involved.

In the adiabatic analysis, the heat transfer is neglected in the working spaces and an isothermal heat exchanger is placed between the working space and the regenerator. This model is valid when the work transfer is much dominant over the heat transfer in the compression and the expansion spaces. It is well-known that the most of the operating conditions for the real Stirling machines are closer to the adiabatic model than the isothermal one.

However, as far as the authors know, the effect of the gas-wall heat transfer in the working spaces on the cycle performance has not been clearly elucidated. One reason is that the gas-wall heat transfer is quite different from the conventional convective heat transfer, as the pressure of the working gas inside the spaces oscillates and the heat transfer may not be in phase with the temperature difference between the gas and the wall. Several experimental and theoretical expressions have been published to date to predict the gas-wall heat transfer and the so-called hysteresis loss for a gas spring. While most of the expressions for the gas spring are not directly applicable to the working spaces of Stirling coolers, some might be useful in predicting its effect on refrigeration.

In this paper, some of the published heat transfer models are incorporated into the first order analysis of Stirling cycle in order to predict how the heat transfer process affects the performance of the cycle. The heat transfer relations to be used here are introduced first. Then the method of the new Stirling cycle analysis is briefly described. Finally, the physical interpretations of the results are presented.

GAS-WALL HEAT TRANSFER MODEL

A number of studies have been performed concerning the heat transfer of an oscillating fluid under oscillating pressures. However, most of the results are not directly applicable to the Stirling cycle analysis, since the situations for the fluid and the heat flow are not the same. On the other hand, some of the results are useful for a rough estimation of the performance of Stirling cycle. Three different functional relationships for the heat transfer in gas spring are introduced, and the results cycle analysis with the heat transfer are compared with the conventional case of the steady flow inside tubes.

Lee-Smith Relation

The very first and the simplest attempt to relate the heat transfer rate with the gas temperature for the gas spring was performed by Lee and Smith. They analytically solved an energy equation without the convection terms to derive an expression for the wall heat flux when the pressure oscillation is first-order harmonic. Their result is given by

$$\dot{Q}(t) = A(t)\frac{k}{D_h}\left[Nu_R(T_{wall} - T(t)) + \frac{Nu_I}{2\pi f}\frac{dT}{dt}\right] \tag{1}$$

where the heat transfer is expressed as a function of the rate of change of the gas temperature as well as the temperature difference between the wall and the gas residing outside of the boundary layer. In Eq.(1), Nu_R and Nu_I are the real and the imaginary parts of a complex Nusselt number, Nu_c, respectively. In their relation Nu_c is defined by

$$Nu_c = \sqrt{\frac{\pi}{2}Pe\frac{D_h}{L_s}}\frac{(1+i)\tanh z}{1-\tanh z/z} \tag{2}$$

where

$$z = (1+i)\sqrt{\frac{\pi}{32}Pe\frac{D_h}{L_s}} \text{ and } Pe = \frac{2\pi f L_s D_h}{\alpha}. \tag{3}$$

When the Peclet number is much smaller than 1, Nu_R is much greater than Nu_I and the second term in Eq.(1) becomes negligible so that the heat transfer is in phase with the temperature difference. As the Peclet number increases, Nu_R and Nu_I tend to become equal in magnitude. It should be noted that the Lee-Smith method is based on a simplified conduction analysis that neglects any convection effect.

Kornhauser-Smith Relation

Kornhauser and Smith performed a series of gas-spring experiments for various gases and operating conditions. From their experimental data, a simple functional relationship for the complex Nusselt number for the case of relatively large Peclet numbers was presented.

$$Nu_R = Nu_I = 0.98\left(Pe\frac{D_h}{L_s}\right)^{0.59} \qquad \text{for} \quad Pe \geq 100 \tag{4}$$

It is noted that Eq. (4) is a quite accurate experimental expression for gas spring and predicts the same behavior as Lee-Smith's for large Pe's.

Jeong-Smith Relation

Jeong and Smith has obtained an approximate numerical solution for the fluid flow and the temperature in a two-dimensional (rectangular) gas spring. The heat transfer rate at the wall was expressed as

$$\dot{Q}(t) = A(t)\frac{k}{H}\left[K_S(T_{wall} - T(t)) + K_T\frac{H^2}{k}\frac{dP}{dt}\right] \tag{5}$$

where

$$K_S = \frac{6H}{\lambda}(1-\zeta)^2 \Big/ \left\{(5-16\zeta+9\zeta^2)+(-19+104\zeta-144\zeta^2+64\zeta^3-5\zeta^4)\left(\frac{6H}{\lambda}\right)\right\} \tag{6}$$

$$K_T = \frac{\lambda}{H}\frac{(1-\zeta)}{(5-\zeta)} \Big/ \left\{\frac{(-5+16\zeta-3\zeta^2)+(25-140\zeta+162\zeta^2-52\zeta^3+5\zeta^4)\left(\frac{6H}{\lambda}\right)}{(5-16\zeta+9\zeta^2)+(-19+104\zeta-144\zeta^2+64\zeta^3-5\zeta^4)\left(\frac{6H}{\lambda}\right)}\right\} \tag{7}$$

$$\zeta \equiv e^{-\frac{H}{\lambda}} \quad \text{and} \quad \lambda(t) = \sqrt{\frac{3kT_{wall}}{\pi f p(t)}\frac{\gamma-1}{\gamma}} \tag{8}$$

In Eq. (8), λ represents the thickness of the thermal boundary layer on the side-wall of the cylinder. This relation is different from Lee-Smith's in that the convection effect is included and also in that the heat transfer is a function of the temperature difference and the rate of change of pressure instead of the rate of change of temperature. This solution is approximate because it is valid only when the stroke is much smaller than the total length of gas spring such that the heat is transferred only through the side wall of the cylinder.

McAdams Relation

The three functional relations just mentioned are derived from the heat transfer models for an oscillating gas spring. For the purpose of comparison, a well-known convective heat transfer relation by McAdams is also introduced here.

$$\dot{Q}(t) = A(t)\frac{k}{D_h}Nu\left(T_{wall} - T(t)\right) \tag{9}$$

where

$$Nu = 0.023\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3} \qquad \text{for } \mathrm{Re} \geq 3000 \text{ and } \mathrm{Re} = \frac{2\pi f L_s D_h}{\nu} \tag{10}$$

Obviously, Eq. (10) is most accurate for steady, fully developed, and turbulent flows in a smooth round tube.

METHOD OF CYCLE ANALYSIS

To illustrate the effects of the heat transfer in the working spaces, the simplest Stirling refrigerator is considered in this study. Figure 1 shows schematically a two-piston refrigerator with typical temperature distribution. For the analysis, it is assumed that the regenerator and the two heat exchangers are perfect without pressure drops and that the working fluid is the helium which behaves like an ideal gas. The direction of heat and work in the heat exchangers and in the compressor and the expander is shown in Figure 1.

The ideal adiabatic analysis for Stirling machines is modified by adding a heat transfer term in the energy balance equation for the compressor and the expander. By combining the mass balance and the energy balance equations, the rate of change of mass in the working spaces can be written as

$$\frac{dm}{dt} = \frac{1}{RT^*}\left\{p\frac{dV}{dt} + \frac{V}{\gamma}\frac{dp}{dt} - \frac{\gamma-1}{\gamma}\dot{Q}\left(T, \frac{dp}{dt}, \frac{dT}{dt}\right)\right\} \tag{11}$$

where T^* is dependent on the direction of flow, and can be defined as the temperature of the gas in the working space if the gas flows out and as the temperature of the adjacent heat exchanger if the gas flows in. The heat transfer rate $\dot{Q}$ is expressed in terms of the gas temperature and the rate of change of temperature or pressure.

For the two heat exchangers and the regenerator, the mass is directly proportional to the pressure, since the temperature does not vary. Once the sum of the five mass change rates is set to zero, the differential equation can be rearranged via the equation of state for an ideal gas such that the pressure change rate is the only unknown. The pressure is integrated while confirming the direction of flow in the working volumes iteratively until a cyclic steady-state is reached.

The final procedure in the analysis is to calculate the heat and the work for a cycle in the working spaces and the heat exchangers by integrating the energy balance equations. The coefficient of performance(COP) for refrigeration is obtained by

$$COP = \frac{Q_L + Q_E}{W_C - W_E} \tag{12}$$

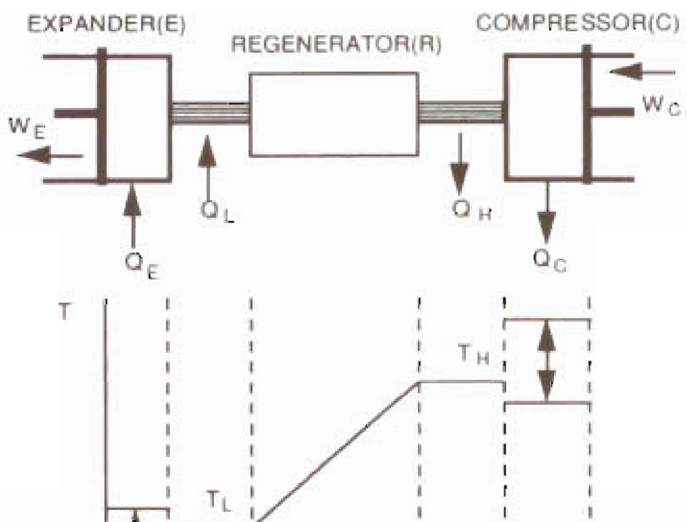

Figure 1. Schematic of two-piston Stirling refrigerator and typical temperature distribution

Table 1 Specifications of simple Stirling cycle for sample calculation

working fluid		He	helium	
total mass		m	5	g
warm temperature		T_H	300	K
cold temperature		T_L	150	K
compressor	swept volume	V_{CM}	1.5	liter
	clearance volume	V_{CC}	0.075	liter
expander	swept volume	V_{EM}	0.75	liter
	clearance volume	V_{EC}	0.0375	liter
bore/stroke		D/L_s	0.55	
phase angle advance of compressor		ϕ	$\pi/2$	rad
dead volume ratio			0.3	

In this study, the 4th-order Runge-Kutta method is used to numerically solve the differential equations. For an appropriate initial value of the pressure, the cyclic steady-state was reached after three or four cycles of iteration.

RESULTS AND DISCUSSION

While the analysis method described above could be applied to the general Stirling cycle, the results are presented here for typical values of design parameters for illustrative purpose. In this study, the volumes of the compressor and the expander have sinusoidal variations given by

$$V_C(t) = V_{CC} + \frac{V_{CM}}{2}\left\{1 + cos(2\pi ft - \phi)\right\} \tag{13}$$

and

$$V_E(t) = V_{EC} + \frac{V_{EM}}{2}\left\{1 + cos(2\pi ft)\right\} \tag{14}$$

respectively. In the working spaces, the total area for the gas-wall heat transfer varies with time and are given by

$$A(t) = \pi D\left(\frac{D}{2} + L(t)\right) \tag{15}$$

which is the sum of the cylinder wall areas and the piston head wall area. It should be noted that the heat transfer in the side wall of the cylinder might be different from that at the top and the bottom walls in general. However, the difference is neglected for the first order approximation.

The basic specifications for the sample calculation are given in Table 1. The dead volume ratio is defined as the total volume of the two heat exchangers plus the regenerator divided by the swept volume of the compressor. It is further assumed that the three dead spaces have the same volume and that the temperature in the regenerator varies linearly along the axial direction. The wall temperatures of the compressor and the expander are taken to be the adjacent heat exchanger temperatures for simplicity.

To validate the analysis method, the pressure variation over a cycle is plotted for various frequencies in Figure 2 including the two limiting cases - isothermal and adiabatic. In the present analysis, the Jeong-Smith relation was used to obtain the heat transfer in the working spaces. It can be clearly observed that the cycle approaches the isothermal limit as the frequency decreases, and that the cycle approaches the adiabatic limit as the frequency increases. In this specific case, the cycle can be considered to be isothermal when the frequency is less than 0.001 Hz, and to be adiabatic when the frequency is greater than 10 Hz. Generally speaking, the dimensionless Nusselt

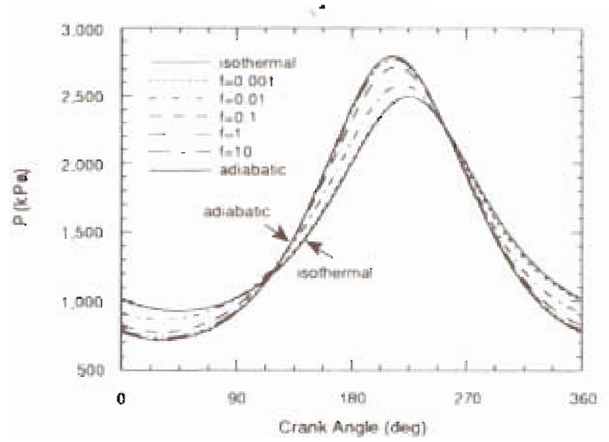

Figure 2. Pressure variation over a cycle for various frequencies for Jeong-Smith model

numbers are functions of Peclet numbers. But it does not imply that the behavior described above is wholly determined by the Peclet numbers, since the heat transfer is dependent upon the operation speed and the heat transfer area as well as the heat transfer coefficient.

Figure 3 shows pressure-volume diagrams of the compressor and the expander for the four heat transfer models when the frequency is set at 0.1 Hz. For this operating condition, the cycles for the two gas-spring heat transfer models (Lee-Smith and Jeong-Smith) are in close agreement. Since the Kornhauser-Smith model is most accurate for large Peclet numbers at high frequencies, a slightly different result is observed. The simple convective heat transfer by McAdams, however, over-estimates the true value since the amplitude of the pressure oscillation is smaller. A more detailed comparison is made in Figure 4. The coefficient of performance in refrigeration was calculated for the four heat transfer models at various frequencies. The dotted curves at low frequencies for the Kornhauser-Smith model and the McAdams model imply that the operating conditions are outside of the applicable ranges. In all cases, as the frequency increases, COP approaches the value of about 0.49, which is COP of an ideal adiabatic cycle. For the Lee-Smith and Jeong-Smith models, as the frequency decreases, COP approaches 1, which is COP of an ideal isothermal or a reversible cycle at the given temperatures. At low frequencies, the higher COP values of the Lee-Smith model compared to the Jeong-Smith model implies correspondingly higher heat transfer rate.

Figures 5, 6, and 7 show the effects of three significant parameters - the phase angle advance of the compressor, the dead volume ratio, and the bore-to-stroke ratio - on the COP of refrigeration, respectively. As the value of the phase angle advance increases, the COP increases, but retains the same value at very low frequencies because it is the reversible asymptote. A similar behavior is observed for the dead volume ratios. As the dead volume ratio increases, the amplitude of the pressure oscillation gets smaller and more entropy generation occurs due to mixing in the working spaces and the irreversible heat transfer in the heat exchangers. On the other hand, the effect of the bore-to-stroke ratio is more complex. Generally, the hydraulic diameter or the characteristic length of the working spaces gets smaller as the ratio increases or decreases from unity. However, for small values of the ratio (the cases of "long" cylinders), the heat transfer area shows high degree of variability, while for large values (the cases of "wide" cylinders), the area does not vary much. Therefore, for large bore-to-stroke ratios, the amount of the overall heat transfer and the COP gets larger.

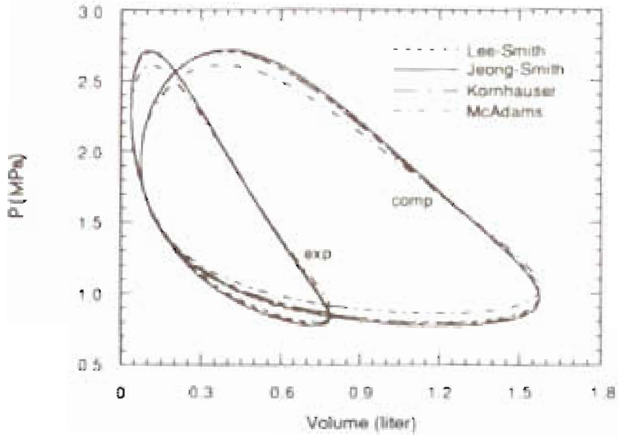

Figure 3. Pressure-volume diagram for four different heat transfer models (f=0.1 Hz)

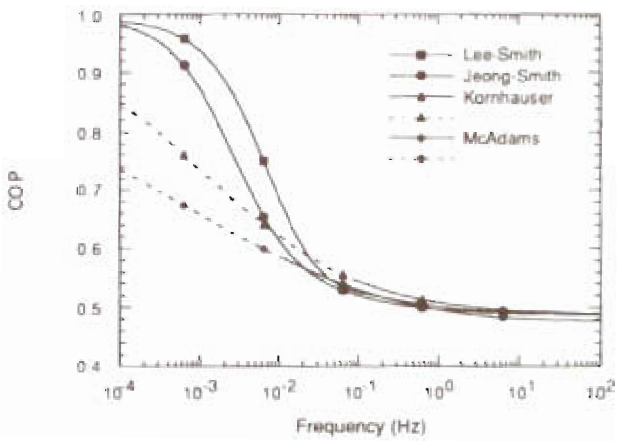

Figure 4. COP vs. frequency for four different heat transfer models

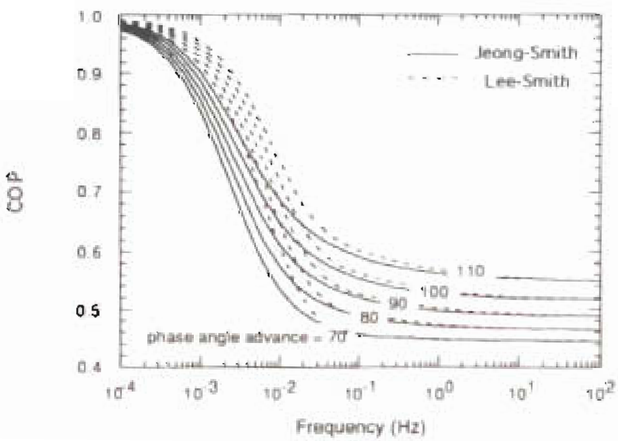

Figure 5. COP vs. frequency for various values of the compressor phase angle advance

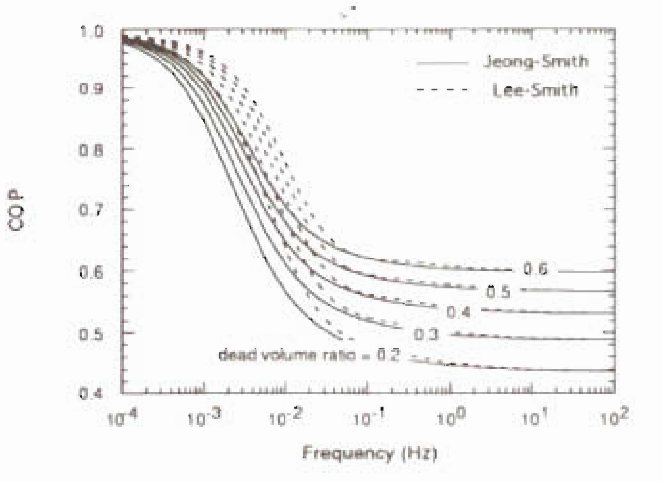

Figure 6. COP vs. frequency for various dead volume ratios

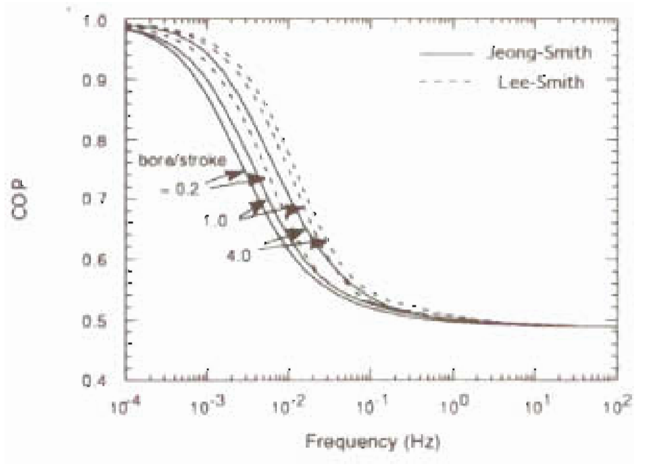

Figure 7. COP vs. frequency for various bore-to-stroke ratios

Finally, it should be mentioned that from the definition of COP, Eq. (12), the heat transfer at the expander can be considered to be the refrigeration in addition to the heat transfer at the cold heat exchanger. In gas-springs, the net gas-to-wall heat transfer is always a dissipation loss or the so-called hysteresis loss, as it is identical to the difference between the (compression) work-in and the (expansion or recovered) work-out. However, the authors think that the issue of whether the heat from the expander is refrigeration or not depends upon one's point of view. If only the heat at the cold heat exchanger is considered to be refrigeration, then the results could be different.

CONCLUSION

A cycle analysis that includes the gas-wall heat transfer in compressors and expanders of Stirling cryocoolers is presented along with previously reported heat transfer models obtained from the gas-spring analyses and experiments. The conventional adiabatic analysis for Stirling cycle is generalized by adding the heat transfer terms to the energy balance equations for the two working spaces. It is found that the effect of the gas-wall heat transfer would not be significant except for low-speed miniature coolers. The analyses are repeated for various operation frequencies, the bore-to-stroke ratios, the dead volumes and the phase angle advances, and their effects on the cooler performance are presented. It can be also mentioned that Jeong-Smith model seems to be most effective in predicting the heat transfer in the working spaces.

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