OPTIMIZATION OF CURRENT LEADS COOLED BY NATURAL CONVECTION OF VAPOR

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ABSTRACT

An optimization theory for current leads passing through a closed vapor-filled space is introduced. These leads are applicable mainly to HTS power systems where liquid nitrogen is continuously refrigerated by a cryocooler. The design of such leads is basically similar to that of conduction-cooled leads, because no boil-off gas flows out of the system. The purpose of this study is to determine whether the cooling by natural convection of vapor requires any modification in optimizing the lead design. In the case that the leads are located in a wide vapor space (called boundary layer flow), the energy balance equations for the lead and the surrounding vapor are solved by the method of perturbation series, as they are weakly coupled by natural convection. The analytical solution shows that the optimal lead parameter does not need to be changed in spite of the convective cooling. In the case of the leads passing through a narrow vapor space (called fully developed flow), on the other hand, the axial conduction of surrounding wall is strongly coupled by natural convection and the lead parameter should be optimized in a modified way. The suggested design conditions are presented in terms of the lead parameter for various RRR values of copper.

INTRODUCTION

Many HTS (high temperature superconductor) power devices are required to use liquid nitrogen as a cooling medium and at the same time to employ a cryocooler for continuous operation. HTS transformers, HTS fault current limiters, and the terminals of HTS transmission cables are some examples under contemporary development [1-4]. Current leads for these closed systems may have a few different options for the cooling method, depending upon the cryostat structure and the technique of electrical insulation.

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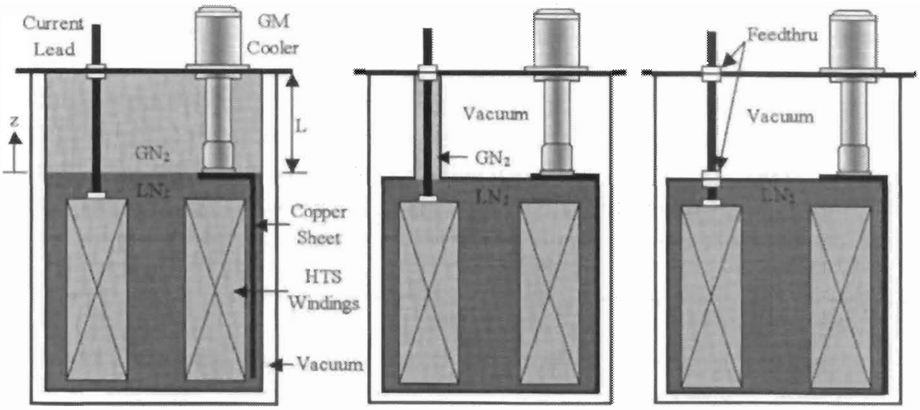

(a) Re-entry cryostat and leads through a wide vapor space (b) Suspended cryostat and leads through vapor-filled neck (c) Suspended cryostat and conduction-cooled leads

FIGURE 1. Schematic of three configurations of zero boil-off cryostat refrigerated by a cryocooler

FIGURE 1 shows schematically three different configurations of a zero boil-off cryostat and the corresponding arrangement of the current leads. The most convenient cryostat for assembly and repeated access to the HTS components is the so-called re-entry type shown in FIGURE 1(a), where the wall of the inner vessel is connected directly to the top flange at room temperature. This type is especially useful when the cryostat is constructed with non-metallic materials as in HTS transformers [2,3]. The leads in these cryostats are located in a wide vapor space above the liquid level.

Heat leak down the wall and through the wide vapor space may be significantly reduced if the inner vessel is suspended by slender supports. For suspended cryostats, we have at least two options for cooling the leads as shown in FIGURE 1(b) and 1(c); the leads may be located in a vapor-filled neck or conductively cooled in vacuum. The main advantage of installing the neck for the leads is that the cold feedthrough required in the conduction-cooled leads may be eliminated.

The optimization of current leads in these three cases is basically similar in that no boil-off gas flows out of the system. In the cases of FIGURE 1(a) and 1(b), however, there exists a certain amount of cooling by natural convection of vapor, as the lead surface always has temperatures higher than the surrounding vapor. This study proposes to answer the questions whether the natural convection in a closed vapor space affects the standard optimization scheme [5,6] and how the penalty caused by the vapor-filled space can be minimized. The two specific cases of FIGURE 1(a) and 1(b), respectively called the boundary layer flow and the fully developed flow, are investigated in this paper.

BOUNDARY LAYER FLOW (WIDE VAPOR SPACE)

Perturbation Method

We first consider the leads passing though a wide vapor space as shown in FIGURE 1(a), where the effect of convective cooling is meager. Since the lead surface in contact with vapor is very small, the convection by the buoyancy force is active only within a boundary layer around the leads. Heat transfer through the vapor outside of the boundary layers is essentially by conduction only, as the top is warmer than the bottom.

The energy balance equations and the boundary conditions can be written in terms of the lead and vapor temperatures as

$$\frac{d}{dz}\left(k_l A_l \frac{dT_l}{dz}\right)+\frac{\rho_l I^2}{A_l}-h_l P_l\left(T_l-T_g\right)=0 \tag{1}$$

$$\frac{d}{dz}\left(k_g A_g \frac{dT_g}{dz}\right)+h_l P_l\left(T_l-T_g\right)=0 \tag{2}$$

$$T_l(0)=T_g(0)=T_L \qquad T_l(L)=T_g(L)=T_H \tag{3}$$

where the vertical distance, z, is measured from the liquid level, and the subscripts, l and g, denote the lead and vapor, respectively. Thermal conductivity, k, and electrical resistivity, ρ, are the temperature dependent properties, and A and P are the cross-sectional area and the axial perimeter, respectively.

The heat transfer coefficient for natural convection on a lead, h_l, is very difficult to calculate or measure with accuracy, mainly because the surface temperature is not axially constant. We predict, however, using a correlation for vertical cylinders [7], that the coefficient is in the range of 4~5 W/m^2-K for typical conditions discussed later. In fact, this value is so small that the conduction through the leads is dominant over the convection to vapor. In other words, the third term in equation (1) plays only a minor role in the energy balance. This implies that the problem may well be solved analytically by the method of perturbation series [8].

The first step is to define a small dimensionless parameter, called the Biot number.

$$\varepsilon \equiv \frac{h_l P_l}{k_l} \ll 1 \tag{4}$$

which is also the condition that justifies the one-dimensional formulation in equation (1). In equation (4), the thermal conductivity of the lead, k_l, can be taken at any reference temperature (for example, at room temperature). Next, the temperatures of the lead and vapor are each expressed as a perturbation series.

$$T_l = T_{l(0)}+\varepsilon\cdot T_{l(1)}+\varepsilon^2\cdot T_{l(2)}+\cdots \tag{5}$$

$$T_g = T_{g(0)}+\varepsilon\cdot T_{g(1)}+\varepsilon^2\cdot T_{g(2)}+\cdots \tag{6}$$

The zeroth-order solutions (or the leading terms in the series) with subscript (0) are the temperatures without the convective cooling, and can be simply obtained by setting $\varepsilon=0$.

$$\frac{d}{dz}\left(k_l A_l \frac{dT_{l(0)}}{dz}\right)+\frac{\rho_l I^2}{A_l}=0 \qquad T_{l(0)}(0)=T_L \qquad T_{l(0)}(L)=T_H \tag{7}$$

$$\frac{d}{dz}\left(k_g A_g \frac{dT_{g(0)}}{dz}\right)=0 \qquad T_{g(0)}(0)=T_L \qquad T_{g(0)}(L)=T_H \tag{8}$$

The ith-order terms ($i\geq 1$) are derived by substituting equations (5) and (6) into equations (1), (2), and (3) and setting the coefficients of ε^i equal to 0. The equations are rearranged in a recurrence form.

$$\frac{d}{dz}\left(k_l A_l \frac{dT_{l(i)}}{dz}\right)=k_l\left(T_{l(i-1)}-T_{g(i-1)}\right) \qquad T_{l(i)}(0)=0 \qquad T_{l(i)}(L)=0 \tag{9}$$

$$\frac{d}{dz}\left(k_g A_g \frac{dT_{g(i)}}{dz}\right) = -k_l\left(T_{l(i-1)} - T_{g(i-1)}\right) \qquad T_{g(i)}(0)=0 \qquad T_{g(i)}(L)=0 \tag{10}$$

where the ith-order solutions can be found by integrating the (i-1)th-order solutions. It is noted in equations (9) and (10) that the boundary conditions are homogeneous and the two ith-order temperatures are not coupled. Even though the number of perturbation terms for a given accuracy may depend on the magnitude of ε, only a few terms are sufficient in most converging problems [8].

Optimization for Wiedemann-Franz Materials

The temperature-dependent properties of lead materials are available from several sources [9,10]. In the temperature range 77~300 K, the properties of copper can be reasonably expressed as simple functions that obey the Wiedemann-Franz law.

$$\rho_l(T)k_l(T) = L_0 \cdot T \qquad \frac{\rho_l(T)}{\rho_l(T_H)} = \left(\frac{T}{T_H}\right)^n \qquad \frac{k_l(T)}{k_l(T_H)} = \left(\frac{T}{T_H}\right)^{1-n} \tag{11}$$

where the Lorenz number, L_0, is taken as the effective value, 2.41 x 10^{-8} W-Ω/K^2 [9], for a better accuracy. For simplicity, we first consider the case of n = 1, then discuss more general cases. If n is taken to be 1, the thermal conductivity of the lead material is constant. If the thermal conductivity of the vapor is also assumed to be a constant, and the perturbation solutions are found in closed-form expressions.

$$T_l(z) = \left[T_H \frac{\sin bz}{\sin bL} + T_L \frac{\sin b(L-z)}{\sin bL}\right] + \varepsilon \cdot T_{l(1)}(z) + O(\varepsilon^2) \tag{12}$$

$$T_g(z) = \left[T_L + (T_H - T_L)\frac{z}{L}\right] - \varepsilon \frac{k_l A_l}{k_g A_g} T_{l(1)}(z) + O(\varepsilon^2) \tag{13}$$

where

$$b = \frac{I\sqrt{L_0}}{k_l A_l} \tag{14}$$

$$T_{l(1)}(z) = \frac{1}{A_l}\left[\frac{T_H}{b^2}\left(\frac{z}{L} - \frac{\sin bz}{\sin bL}\right) + \frac{T_L}{b^2}\left(1 - \frac{z}{L} - \frac{\sin b(L-z)}{\sin bL}\right)\right.$$
$$\left. + \frac{T_L}{2}(L-z)z + \frac{(T_H - T_L)}{6}(L^2 - z^2)\frac{z}{L}\right] \tag{15}$$

The higher order terms are omitted due to space limitation.

The sum of the heat leaks through the lead and vapor per unit current can be obtained from equations (12) and (13),

$$\frac{Q_{l+g}(0)}{I} = \sqrt{L_0}\frac{T_H - T_L \cos bL}{\sin bL} + \frac{k_g A_g}{IL}(T_H - T_L) + O(\varepsilon^2) \tag{16}$$

The first-order terms have been eliminated, as the decrease of the cooling load for the lead is identical to the increase of the cooling load for the vapor. In other words, the amount of heat rejected from the lead is eventually delivered to the cold end through the vapor.

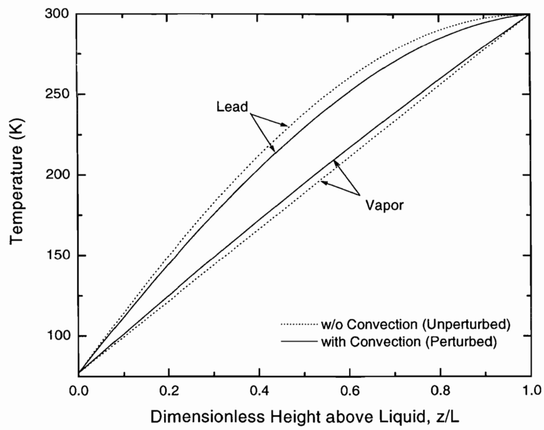

FIGURE 2. Vertical temperature distributions for the leads and vapor without (unperturbed) and with (perturbed) natural convection of vapor. ($bL = 1.331$, $h_l P_l / k_l = 0.000428$, $k_l A_l / k_g A_g = 1.37$)

The optimal condition to minimize the cooling load can be found from equation (16)

$$\cos bL \approx \frac{T_L}{T_H} \quad \text{or} \quad \left(\frac{IL}{A_l}\right)_{opt} \approx \frac{k_l}{\sqrt{L_0}} \cos^{-1} \frac{T_L}{T_H} \tag{17}$$

which is exactly the same as for the conduction-cooled leads [5,6]. The key point of this result is that the individual loads for the lead and the vapor are affected by convection, but the sum of the loads is not, insofar as they are weakly coupled.

FIGURE 2 shows an example of the axial temperature distributions of the lead and vapor, with and without the perturbation terms. In the calculation, the dimensionless parameter, bL, is taken as 1.311, which is an optimum for $T_H = 300$ K and $T_L = 77$ K from equation (17). By adding the effect of the convection as perturbation terms, the difference between the two temperatures is reduced, and the temperature gradient at $z = 0$ becomes smaller for the lead, but larger for the vapor.

The optimal condition and the corresponding load can now be generalized for the temperature-dependent conductivities [5,6]:

$$\left(\frac{IL}{A_l}\right)_{opt} = \int_{T_L}^{T_H} \frac{k_l}{\sqrt{L_0 \left(T_H^2 - T^2\right)}} dT \tag{18}$$

$$\left(\frac{Q_{l+g}(0)}{I}\right)_{\min} = \sqrt{L_0 \left(T_H^2 - T_L^2\right)} + \frac{A_g}{IL} \int_{T_L}^{T_H} k_g dT + O\left(\varepsilon^2\right) \tag{19}$$

FULLY DEVELOPED ASYMPTOTE (VAPOR-FILLED NECK)

The other extreme case considered here is an asymptote that the leads pass though a vapor-filled neck as shown in FIGURE 1(b) or the effect of convective cooling is very significant. In this case, the thickness of boundary layer is greater than the gap between the lead and the neck wall, so the vapor in the narrow gap generates a fully developed (or circulating) flow by natural convection. The energy balance equations and the boundary conditions for the lead and the neck wall (subscript w) are written as

$$\frac{d}{dz}\left(k_l A_l \frac{dT_l}{dz}\right)+\frac{\rho_l I^2}{A_l}-h_l P_l\left(T_l-T_w\right)=0 \tag{20}$$

$$\frac{d}{dz}\left(k_w A_w \frac{dT_w}{dz}\right)+h_l P_l\left(T_l-T_w\right)=0 \tag{21}$$

$$T_l(0)=T_w(0)=T_L \qquad T_l(L)=T_w(L)=T_H \tag{22}$$

where h_l is the surface-to-surface heat transfer coefficient. In contrast to the previous case, the two solid temperatures are strongly coupled by active heat transfer.

In the limit of fully effective heat transfer, the two temperatures become identical, and the energy balance can be simplified by adding equations (20) and (21) and setting $T_l \approx T_w \approx T$ to get

$$\frac{d}{dz}\left[k_l A_l (1+\varepsilon)\frac{dT}{dz}\right]+\frac{\rho_l I^2}{A_l}\approx 0 \qquad T(0)=T_L \qquad T(L)=T_H \tag{23}$$

where

$$\varepsilon \equiv \frac{k_w A_w}{k_l A_l} \tag{24}$$

Equation (23) has the same shape as the energy balance of the conduction-cooled leads, except that there is an additional cross-sectional area for heat conduction. Therefore the optimal condition and the corresponding load can be expressed approximately by including the effect of the additional area as a constant. For the Wiedemann-Franz materials,

$$\left(\frac{IL}{A_l}\right)_{opt} \approx \int_{T_L}^{T_H} k_l \sqrt{\frac{1+\varepsilon}{L_0\left(T_H^2-T^2\right)}}\,dT \tag{25}$$

$$\left(\frac{Q_{l+w}(0)}{I}\right)_{\min} \approx \sqrt{L_0(1+\varepsilon)\left(T_H^2-T_L^2\right)} \tag{26}$$

In comparison with the conduction-cooled leads, the optimal lead parameter is greater by the factor of $\sqrt{1+\varepsilon}$, and the cooling load is increased by the same factor according to the penalty of installing the neck.

DISCUSSION

FIGURE 3 is a plot of the cooling load per unit current as a function of the lead parameter (*IL/A*) for various cases, demonstrating the significance of the present analysis. As a reference, the curve at the bottom is for the conduction-cooled leads ($\varepsilon = 0$). As is well known, the lead parameter has a unique optimum indicated by the dot, which is 34.1 kA/cm for copper leads with *RRR* = 60, and the corresponding cooling load (i.e. the minimum) is 42.0 W/kA.

The top curve noted by "boundary layer flow" is the cooling load of the leads and vapor, when one pair of 200-A leads pass through a vapor space with diameter of 0.95 m and a height of 0.4 m. As also indicated by the dot, the optimal lead parameter is the same

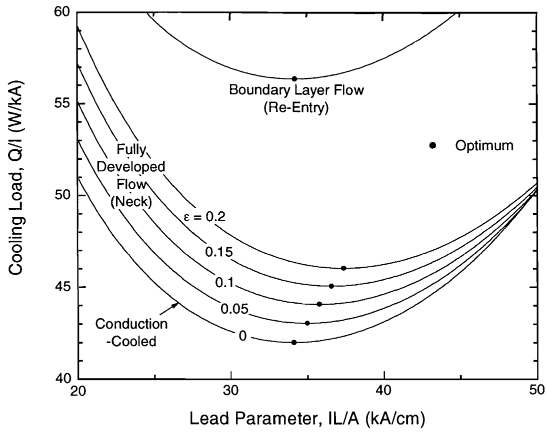

FIGURE 3. Cooling load of leads per unit current as a function of the lead parameter (*IL/A*) for boundary layer flow and fully developed flow in comparison with conduction-cooled leads. (Copper with *RRR* = 60, T_H = 300 K, T_L = 77 K)

(34.1 kA/cm) as the conduction-cooled leads, even though the minimum load may be greater by 14.4 W/kA. It is clear from equation (19) that this difference in the cooling loads is the conduction through the vapor. However, the actual heat from vapor to liquid must be greater than this value because of the convection from the lead, but the cooling load of the leads are also smaller by the same amount as explained earlier by FIGURE 2. It is also important in a practical load calculation to add the heat conduction through the wall of liquid vessel, but we will not discuss this further, because the thermal coupling of the wall and vapor by convection is even weaker and the addition of the conduction load is trivial.

A group of curves noted by "fully developed flow" are the cooling loads of the leads with the vapor-filled neck, for various values of ε defined by equation (24). As ε increases, the contribution of the neck conduction becomes more and more important so that both the cooling load and the optimal lead parameter yielding the minimum load tends to have a greater value. The key point here is that the increase of the cooling load, or the penalty of installing the neck can be reduced by a modified lead parameter. The magnitude of the reduction does not seem to be considerable from FIGURE 3, yet the optimal design is still of significant value in preparation for any off-design operation. At the same time, these quantitative results should be useful in the design consideration of trade-off between the practical convenience and the absolute minimum of cooling load.

Based upon the optimization theory described above, FIGURE 4 presents the optimal lead parameter as a function of liquid temperature for various RRR values of copper, as some of the HTS power devices are operated at temperatures lower than 77 K.

CONCLUSIONS

The questions regarding the current leads passing through a closed, vapor-filled space are clearly answered through this study. When the leads are located in a wide vapor space or the natural convection is characterized by boundary layer flow, the optimal lead parameter is the same as for the conduction-cooled leads, but the total cooling load is

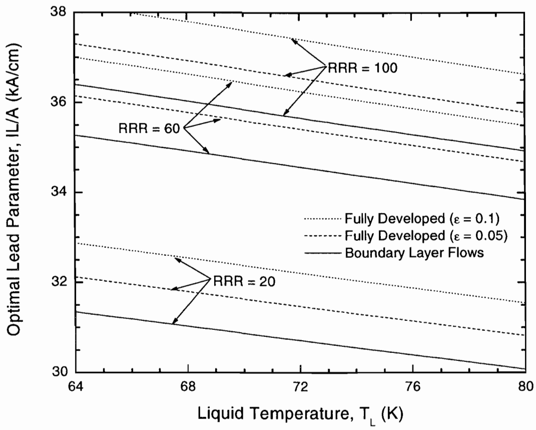

FIGURE 4. Optimal lead parameter as a function of liquid temperature with various RRR values of copper for boundary layer flow and fully developed flow.

higher. When the leads pass through a narrow vapor space, or the lead temperature is strongly coupled with the nearby wall by fully developed flow, the cooling load can be closer to the pure conduction-cooled leads, but the optimum lead parameter must be modified.

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