OPTIMIZATION OF TWO-STAGE CONDUCTION COOLING FOR HTS CURRENT LEADS

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ABSTRACT

An optimal two-stage cooling method for HTS current leads is developed to minimize the required refrigerator power input. The lead is a series combination of a normal metal conductor at the warmer part and an HTS at the colder part. The cold end of the HTS lead is conduction-cooled by a cryocooler and heat is also intercepted at an intermediate axial location. Mathematical expressions are derived for the required refrigerator power per unit current for typical cryocoolers. It is demonstrated that both the current density and the intercept temperature should be optimized simultaneously to minimize the refrigerator power per unit current and maintain the HTS in a superconducting state. In addition, it is verified that heat should be intercepted at the joint of the two parts or at an optimal axial position of the metal lead for the minimum refrigerator power. The design conditions for the two-stage cooling are quantitatively presented with the effects of the magnetic field and the stability margins of the HTS lead, when the cold end temperature is 4 K or 30 K.

INTRODUCTION

High Tc superconductors(HTS) are gradually replacing conventional metallic current leads in high field superconducting magnets, because they can reduce considerably the heat leak to the cryogenic temperatures and the required refrigerator power. The so-called binary lead is a series combination of a normal metal conductor at the warmer part and an HTS at the colder part. The cooling method for the binary lead may be quite different from the standard helium-vapor cooling of metal leads,[1-3] and a number of studies have been performed during past several years for an effective cooling technology of the bulk or the composite type of HTS leads.[4-11]

Conduction cooling of current leads has received close attention from many researchers since the successful development of 4 K Gifford-McMahon cryocoolers and liquid-free superconducting systems.[12] There is no liquid cryogen in the systems, so the lead should be cooled in vacuum by direct contact with cryocoolers. In addition, conduction cooling of

HTS leads is also applied to HTS magnet systems[13] where a high field is required and the operation temperature is lower than about 30 K.

The optimization techniques of conduction cooling for metal leads were studied earlier,[1,2] including single- or multi-stage (even distributed) methods. In these methods, the geometric conditions of the lead were designed such that the heat conduction and the heat generation may be balanced or the total refrigerator power required for the cooling loads may be minimized. Cooling of HTS leads without boil-off helium gas has been partly considered in some publications.[4-7] Most of these research works are, however, related with the analysis or the experiment for HTS leads whose ends were cooled by liquid nitrogen or liquid helium. Two recent papers[8,9] have tried to find the optimal intercept temperature for a given geometric condition of the binary lead.

The present author has lately published a new and generalized theory[10] to reveal that both the operating current density and the intercept temperature of binary lead should be optimized simultaneously for the most efficient cooling. The theory also provided the absolute minimum in refrigerator power for the binary lead as a thermodynamic limit for distributed or two-stage conduction cooling. He has also extended his original theory to a practical design tool by incorporating the performance of some actual cryocoolers and the realistic stability margins of the HTS[11].

This paper aims at a complement of the optimization technique for the two-stage conduction cooling by considering three additional factors that have not been included in the previous studies. The first is a theoretical consideration in association with the axial location of the heat intercept. In every previous study about two-stage cooling of binary leads, heat is intercepted at the joint of the metal and the HTS leads. A question has been raised how the refrigerator power would vary if the intercept location is not the joint. The second is a more practical and quantitative examination for the effect of the magnetic field on the optimal cooling conditions of the leads. Finally, the optimization theory is applied to binary leads for HTS magnet systems whose operating temperature is as high as 30 K, as the previous studies treated only the leads for the standard low Tc systems at 4 K.

OPTIMIZATION METHOD

Figure 1 shows schematically the two-stage conduction cooling of a binary current lead. The HTS part and the normal metal part in the binary lead are denoted by subscripts 1 and 2, respectively. Heat, Q_L, is removed from the cold end whose temperature is T_L, and Q_I is intercepted at an intermediate axial location where the temperature is T_I. T_J is the temperature at the joint of the two parts and identical to T_I in case of the intercept at joint as shown in (a). If heat is intercepted at a certain point on the metal or on the HTS, T_J would be smaller than T_I or greater than T_I as in (b) or (c), respectively. The warm end temperature, T_H, is the room temperature, and L is the length of the leads.

Required Refrigerator Power

The basic concept of the thermodynamic optimization for the two-stage cooling is to minimize the total required refrigerator power per unit current in steady-state. Since there exist two cooling loads at two different temperatures, the total refrigerator power per unit current can be expressed in general as

$$\frac{W_{ref}}{I} = \frac{Q_L}{I} \cdot \frac{1}{COP_L} + \frac{Q_I}{I} \cdot \frac{1}{COP_I} \tag{1}$$

where COP's are the coefficients of performance of the cryocooler at each stage.

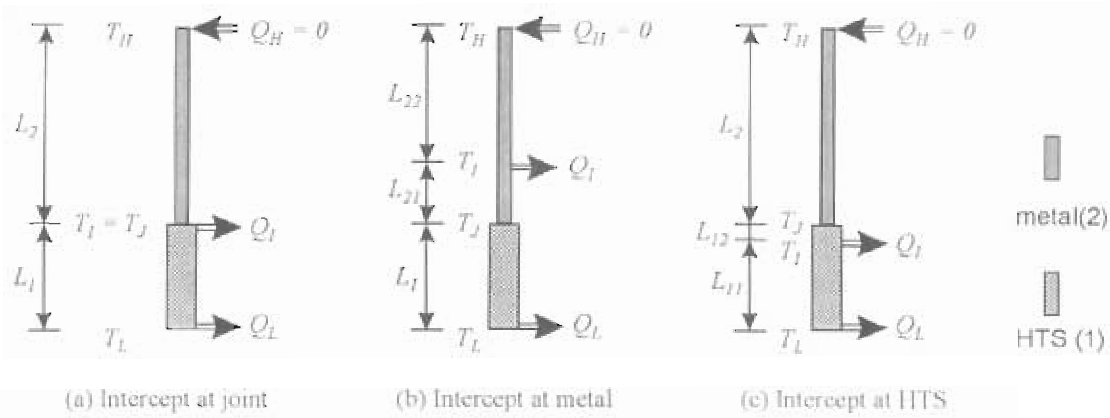


Figure 1. Schematic representation of two-stage conduction cooling method of a binary (metal + HTS) current lead with three different intercept locations.

Since the HTS lead should be always superconducting in normal operation, the current density of the HTS, J_1, cannot exceed its critical value, J_C. The critical current density of a superconductor is a strong function of the temperature and the magnetic field, B. Thus Eq. (1) should be minimized with a constraint,

$$J_1 \leq J_C(T_J, B) \tag{2}$$

because the highest temperature in the HTS lead is T_J.

Cryocooler Performance

In order to calculate the required refrigerator power in Eq. (1), the coefficient of performance should be known for the specific cryocoolers to be used. In general, the COP of a cryocooler depends on the refrigeration temperature, the temperature at which heat is rejected, the type of refrigeration cycle, the cooling capacity, the performance of its components, and so on. For the purpose of a more quantitative demonstration of the optimization in this paper, simple, yet reasonable, COP models are selected from the previous study[11]. Figure 2 shows the COP's as a function of refrigeration temperature for medium or small sizes of various commercial cryocoolers, from which two simple mathematical functions have been fitted as the dashed curves or

$$COP_L = \frac{T_L}{T_H - T_L} \cdot \frac{0.1202 \cdot T_L + 1.316}{T_L + 84.81} \qquad 4\,\mathrm{K} \leq T_L \leq 40\,\mathrm{K} \tag{3}$$

$$COP_I = \frac{T_I}{T_H - T_I} \cdot \frac{0.4198 \cdot T_I + 2.740}{T_I + 1115} \qquad 20\,\mathrm{K} \leq T_I \leq 100\,\mathrm{K} \tag{4}$$

where $T_H = 300$ K.

Intercept Location and Cooling Loads

Two cooling loads per unit current in Eq. (1) can be expressed in terms of the temperature-dependent properties of the lead materials and the dimensional factors in each case of the intercept location. If the heat generation in the HTS is assumed to be zero and heat is intercepted at the joint,

$$\frac{Q_L}{I} = \frac{1}{J_1 L_1} \int_{T_L}^{T_J} k_1 \cdot dT \tag{5}$$

$$\frac{Q_J}{I} = \sqrt{2 \int_{T_J}^{T_H} k_2 \rho_2 \cdot dT} - \frac{Q_L}{I} \tag{6}$$

where k and ρ are the thermal conductivity and the electrical resistivity, respectively. In Eq. (6), the first term of the right-hand side is the minimum heat transfer rate per unit current from the metal part to the joint, which can be obtained when the dimension of the lead is optimized so that the conduction and the heat generation may be balanced or $Q_H = 0$.[1,10]

In case of heat intercept at a certain axial location of the metal lead, the metal lead is divided into two segments. Each segment can be optimized in the same manner as in Eq. (6), but with a different temperature span. Then, since the heat flow at the joint should be continuous, the two cooling loads per unit current can be expressed as

$$\frac{Q_L}{I} = \frac{1}{J_1 L_1} \int_{T_L}^{T_J} k_1 \cdot dT = \sqrt{2 \int_{T_J}^{T_I} k_2 \rho_2 \cdot dT} \tag{7}$$

$$\frac{Q_I}{I} = \sqrt{2 \int_{T_I}^{T_H} k_2 \rho_2 \cdot dT} \tag{8}$$

It should be noted that the length of each segment of the metal lead is not shown in Eq. (7) or (8), because it has already been optimized.

Even if the intercept at the HTS may not seem to be a practical option, the two cooling loads in case of Figure 1(c) can be theoretically obtained on an assumption that the HTS is superconducting and the dimension of the metal lead is optimized.

$$\frac{Q_L}{I} = \frac{1}{J_1 L_{11}} \int_{T_L}^{T_I} k_1 \cdot dT \tag{9}$$

$$\frac{Q_I}{I} = \frac{1}{J_1 L_{12}} \int_{T_I}^{T_J} k_1 \cdot dT - \frac{Q_L}{I} = \sqrt{2 \int_{T_J}^{T_H} k_2 \rho_2 \cdot dT} - \frac{Q_L}{I} \tag{10}$$

where L_{11} and L_{12} are the lengths of each segment of the HTS lead as shown in Figure 1(c).

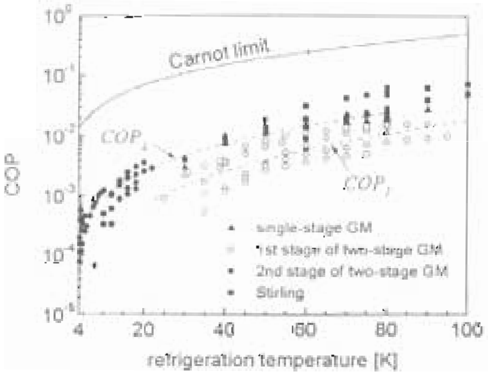


Figure 2. Coefficient of performance of various cryocoolers as a function of refrigeration temperature with the Carnot limit. COP_L and COP_I from Eq. (3) and (4) are shown by dashed curves.

Critical Current Density of HTS

The critical property of the HTS lead plays an important role as a constraint, Eq. (2), in the optimization. Again for the purpose of a more quantitative demonstration of the method, a reasonable behavior is assumed for the critical current density of HTS. For a bulk Bi-2223, it is reasonably accurate that J_C is a simple function[4,7] of temperature and magnetic field,

$$J_C(T_J, B) = 4 \cdot J_C(77\,\mathrm{K}, 0\,\mathrm{T}) \cdot \frac{1+B}{1+5.5B} \cdot \left[1 - \frac{T_J}{104}\right] \qquad (T_J \text{ in K}, B \text{ in Tesla}) \tag{11}$$

where $J_C(77\ K,\ 0\ T)$ is the characteristic critical current density at 77 K and 0 T, depending upon the size, the shape and the fabrication method.

Stability Margin of HTS

Because of the nature of the superconductivity, the required refrigerator power has its minimum always at a marginally superconducting state[10,11] by the constraint, Eq. (2). In practice, the HTS current lead should be designed such that the current density is well below the critical value in order to be stable from a certain level of thermal disturbance. One of the design schemes to determine the operational current density as a fraction, ϕ, of the critical value.

$$J_1 = \phi \cdot J_C(T_J, B) \tag{12}$$

RESULTS AND DISCUSSION

Optimal Design

The refrigerator power per unit current, W_{ref}/I, is calculated by substituting Eq. (3) through (6) into Eq. (1), in case of the intercept at joint. When the materials of the lead and the two end temperatures are given, it is noted that W_{ref}/I is a function of J_l, L_l and T_J only. Figure 3(a) shows some contours of constant W_{ref}/I on a J_l-T_J diagram for Cu + Bi-2223 lead when L_l = 20 cm and T_L = 4 K. In the calculation, the properties of the materials are taken from two recent publications.[3,6] It is a general trend that W_{ref}/I decreases as J_l or T_J increases. However, if T_J is low or J_l is high, W_{ref}/I is almost independent of J_l, since the cooling load for the HTS is relatively small. On the contrary, if T_J is high or J_l is low, the cooling load for the HTS is relatively significant and J_l is a more important factor than T_J.

As mentioned above, the minimization of W_{ref}/I is subject to a constraint, Eq. (2). Figure 3(a) shows critical current density, J_C, as a function of T_J, Eq. (11), assuming that J_C (77 K, 0 T) = 1,000 A/cm^2 and B = 0 T. Clearly, there exists unique optimal values for J_l and T_J to minimize W_{ref}/I as indicated by a dot in a marginally superconducting state. It is also noted that if J_l is given as a constant, the optimal T_J to minimize W_{ref}/I may be significantly lower than T_C because of the concave shape of the contour curves, as discussed in other publications.[8,9] Therefore the present method in this paper can be considered as a two-dimensional optimization involving the previous one-dimensional method as a special case.
The theoretical minimum of W_{ref}/I in this case is approximately 3.09 W/A, which is about 51 times the absolute minimum as a thermodynamic limit[10] that can be achieved by the most efficiently distributed Carnot refrigerators. The corresponding optimal values of J_l and T_J are approximately 378 A/cm^2 and 94.2 K, respectively. The stability margin of the HTS can be included as in Eq. (12) for a practical design. Two examples for ϕ = 0.5 and 0.2 are indicated in Figure 3 and the required minimum for W_{ref}/I is increased to about 3.54 W/A

and 4.46 W/A, respectively. It is obvious that J_I and T_J should be smaller than the respective theoretical optima.

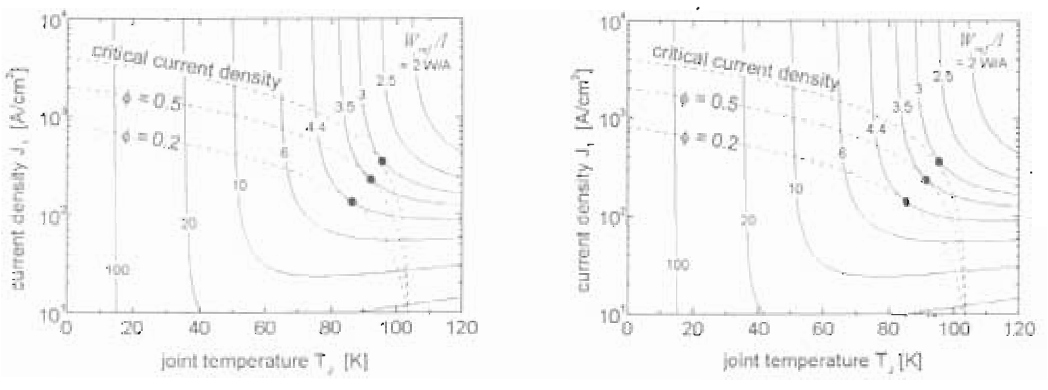
When heat is intercepted at an optimal location of the metal lead, the required W_{ref}/I is calculated by substituting Eq. (7) and (8) into Eq. (1). In this case, W_{ref}/I is explicitly a function of J_I, L_I, T_J and T_I for the given materials and the end temperatures. However, T_J and T_I are not independent because of Eq. (7), thus W_{ref}/I can be expressed as an implicit function of J_I, L_I, and T_J again. Figure 3(b) shows contours of W_{ref}/I on a J_I-T_J diagram in this case with the same conditions as (a). The values of W_{ref}/I are indistinguishable from those in case (a), except for the very low T_J region. It can be concluded that the minimum W_{ref}/I and the optimal conditions for J_I and T_J would be almost the same as the case of the intercept at joint, as far as each segment of the metal lead is optimally dimensioned.

The intercept at a certain location of the HTS lead may not be suitable both in its hardware construction and in its cooling, because of the huge difference in the thermal conductivity between the metal and the HTS. Just for a thermodynamic analysis, the required W_{ref}/I can be calculated by substituting Eq. (9) and (10) into Eq. (1). In this case, W_{ref}/I is an implicit function of L_{I1}, L_{I2}, J_I and T_J, since T_I are connected with T_J by Eq. (10). For a fair comparison, W_{ref}/I should calculated when the total length of the HTS lead is given as the same value or $L_I = L_{I1} + L_{I2}$. It can be verified, however, that the W_{ref}/I is a monotonously increasing function of L_{I2} and therefore has its minimum at $L_{I2} = 0$ for a given L_I. Obviously, there is no advantage for the intercept at any location of the HTS.

Effects of Magnetic Field and Cold End Temperature

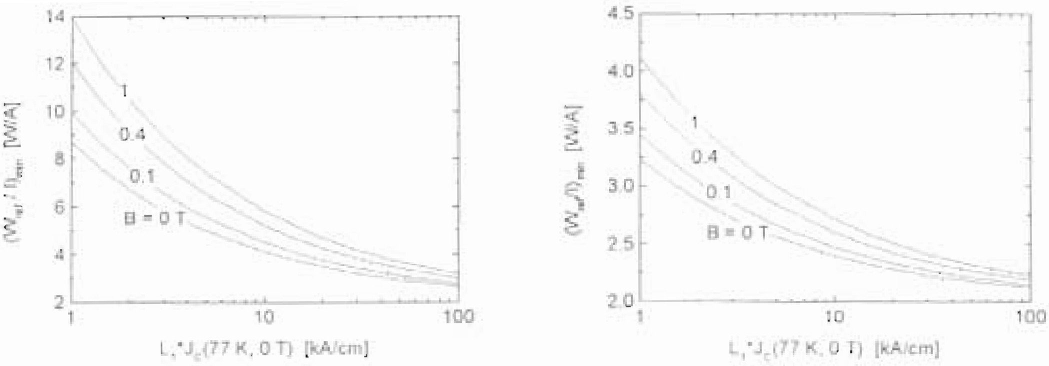
As demonstrated in the previous section, the unique minimum in W_{ref}/I is obtained from the unique optima of J_I and T_J, for given L_I, J_C (77 K, 0 T), T_L, B and ϕ, if the cryocooler and the lead materials are given. It has been proven from a dimensional analysis[11] that the effect of L_I on the minimum W_{ref}/I is exactly the same as that of J_C (77 K, 0 T), so the product of L_I and J_C (77 K, 0 T) can be a single parameter in the optimization.

$$\left(\frac{W_{ref}}{I}\right)_{\min} = f\left[L_1 \cdot J_C(77\,K, 0\,T),\ T_L,\ B,\ \phi\right] \tag{13}$$



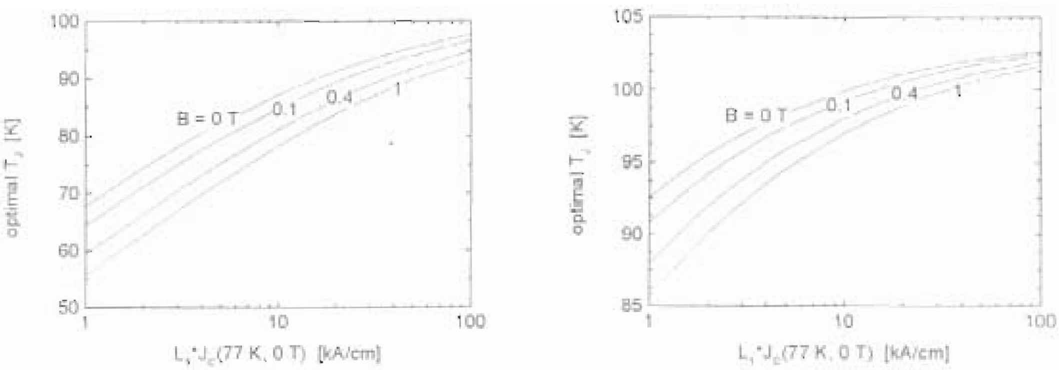
(a) Intercept at joint (b) Intercept at metal

Figure 3. Contours of refrigerator power per unit current on current density of HTS vs. joint temperature (J_I - T_J) diagram of Cu+Bi2223 lead when L_I = 20 cm and T_L = 4 K. Dots indicate the optimal conditions to minimize the refrigerator power per unit current.



(a) T_L = 4 K (b) T_L = 30 K

Figure 4. Minimum refrigerator power per unit current as a function of the product of the critical current density and the length of HTS for Cu+Bi2223 lead when ϕ = 0.5.



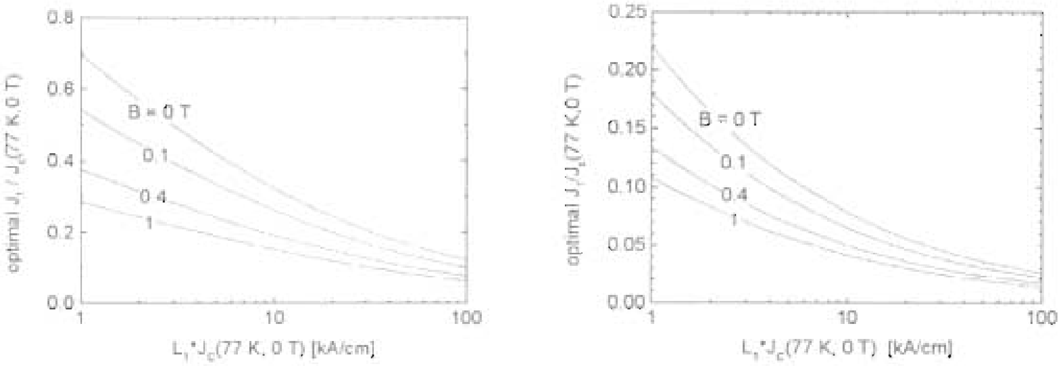
(a) T_L = 4 K (b) T_L = 30 K

Figure 5. Optimal intercept temperature as a function of the product of the critical current density and the length of the HTS for Cu+Bi2223 lead when ϕ = 0.5.

The above optimization procedure has been repeated for various values of $L_1 \cdot J_C(77\,\mathrm{K}, 0\,\mathrm{T})$, T_L and B, and the some results for $\phi = 0.5$ have been plotted in Figures 4 through 6. The minimum of W_{ref}/I decreases as $L_1 \cdot J_C(77\,\mathrm{K}, 0\,\mathrm{T})$ increases or B decreases. However, W_{ref}/I does not vary significantly when $L_1 \cdot J_C(77\,\mathrm{K}, 0\,\mathrm{T})$ is greater than 20 kA/cm, which means for example that L_l >20 cm if J_C (77 K, 0 T) = 1,000 A/cm^2 or L_l >10 cm if J_C (77 K, 0 T) = 2,000 A/cm^2. It is noted that the length and the critical current density play exactly the same role in the cooling from the thermodynamic point of view, while the magnetic field is related in a more complicated manner with the critical current density as in Eq. (11). If $L_1 \cdot J_C(77\,\mathrm{K}, 0\,\mathrm{T})$ decreases to zero, W_{ref}/I for B = 0 approaches to its limit of the single-stage cooling of an optimized metal lead at the cold end.[11] On the contrary, if $L_1 \cdot J_C(77\,\mathrm{K}, 0\,\mathrm{T})$ is infinitely large, Q_L vanishes and W_{ref}/I would have the value

$$\frac{W_{ref}}{I} = \frac{1}{COP_I(T_C)} \sqrt{2 \int_{T_C}^{T_H} \rho_2 k_2 \cdot dT} \tag{14}$$

where T_C is the critical temperature of HTS. The minimum W_{ref}/I for T_L = 30 K is expected to be about 30 or 70% of that for T_L = 4 K, depending on B. As $L_1 \cdot J_C(77\,\mathrm{K}, 0\,\mathrm{T})$ increases or B decreases, the optimum for J_l increases and the optimum for T_J decreases. The optimal J_l for T_L = 4 K is higher by 5 to 30 K than that for T_L = 4 K in these conditions.



(a) T_L = 4 K (b) T_L = 30 K

Figure 6. Optimal operating current density of HTS as a function of the product of the critical current density and the length of HTS for Cu+Bi2223 lead when ϕ = 0.5.

CONCLUSIONS

A comprehensive optimization technique of two-stage conduction cooling is presented for the metal-HTS current lead with some quantitative design data. The required refrigerator power per unit current for two-stage cooling is minimized by optimizing the dimensions of the lead and the heat intercept stage if the end temperatures and the properties of the lead materials are given. The procedure is demonstrated with every factor to be included in practical design, such as the refrigeration performance of the actual cryocoolers, the temperature dependent properties of the materials, the effect of magnetic field on the critical current density of HTS, and the stability margin of the HTS.

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